Learning and Strategic Trading in ETF Markets*

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Abstract

Designated broker-dealers arbitrage away differences between the market price of an ETF and the net asset value of the underlying assets. Using a dynamic strategic trading model, I show that this arbitrage mechanism increases long-term price informativeness but reduces short-term price informativeness. The information contained in the ETF price leads to additional learning, which improves long-term price informativeness. However, traders informed about the value of an underlying asset use their informational advantage to forecast arbitrage-induced price changes of all other assets contained in the ETF. The predictability of future price changes induces speculative cross-asset trading, which reduces short-term price informativeness. Thus, regulation targeting ETFs must balance short- and long-term price informativeness.

Keywords: ETF, Market efficiency, Strategic trading, Information asymmetry, Learning

JEL Codes: G1, G14, D4, D82, D83

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The assets managed by exchange traded funds (ETFs) have grown remarkably over the past years. As of Q2 2023, ETFs manage around \$5.8 trillion of equities in the U.S. (Statista 2023). ETFs not only offer access to diversification, but are also highly liquid and have a low tracking error due to their institutional design. Designated broker-dealers—so called authorized participants (AP)—can continuously create / redeem ETF shares in exchange for the basket of underlying securities (see Appendix A). Being exchange traded, the price of an ETF depends on the ETF's order flow. Authorized participants continuously arbitrage differences between the price and net asset value of an ETF away. The simultaneous buying / selling of the basket of underlying securities leads to correlated order flows for the underlying assets, with potentially ambiguous effects on the prices of the underlying assets. Some studies suggest that ETFs increase price informativeness (Hasbrouck 2003, Glosten et al. 2021, Buss and Sundaresan 2023), while others find a detrimental effect on price informativeness (Agarwal et al. 2018, Ben-David et al. 2018, Todorov 2019).

In this paper, I show that the institutional design of ETFs leads to a reduced short-term price informativeness and an increased long-term price informativeness. The price of the ETF provides a signal about the weighted average value of the underlying assets. Speculators who are informed about the value of one underlying asset can predict price changes of the ETF as well as the arbitrage activity of the APs. Making rational use of their forecast, speculators trade the assets contained in the ETF to (a) protect their informational advantage vis-à-vis the market makers that price the underlying asset and to (b) profit from price changes due to APs arbitrage activity. However, because the trading is non-informational, it worsens short-term price informativeness. In contrast, ETFs increase long-term price informativeness because market participants use the additional information provided by the ETF price to learn about the value of the underlying assets. The results in this paper suggest that any regulation targeting ETFs must balance effects on short-term and long-term price informativeness.

Model setup. I analyze a two-period Kyle (1985)-model of strategic trading in two assets with independent fundamental values. Each asset is traded in a separate but accessible market by an informed speculator and a mass of uninformed traders that buy or sell the asset for liquidity reasons. Each informed speculator knows the fundamental value of one asset. Competitive and rational market markers set the price for each asset.

On top of the two underlying assets, I introduce an ETF that contains both assets. The ETF is traded and priced in a segmented market, whereby traders are exogenously assigned to trade either the underlying assets or the ETF. This assumption allows me to cleanly identify the informational effects that emerge from ETFs, without a confounding influence arising from the additional trading opportunity. Moreover, Boulatov et al. (2013) and Cespa and Colla (2020) show that segmented markets considerably simplify the optimization of the informed speculator.

The arbitrage mechanism underlying ETFs implies that price discovery and information flows occur in stages. At $t = 1$, the assets and the ETF are traded simultaneously and independently. The prices of the underlying assets and the ETF only reflect the information contained in the contemporaneous order flow in the respective market. After prices are set, at $t = 1.5$, APs arbitrage price differences between the basket of underlying securities and the ETF away and provide an additional source of information to all market makers. Market makers use this additional information and update prices accordingly.[1] Afterwards, another trading period ($t = 2$)—similar to the first trading period—begins.

Findings. I show that ETFs introduce a new cross-market learning opportunity. Intuitively, the price of the ETF contains information about the weighted average value of the underlying securities and this additional information is used efficiently by the speculators and market makers. To fix ideas, let us call the two underlying assets a and b, and consider the market maker that is pricing asset a. At $t = 1$, the market maker observes the order flow for asset a and sets the semi-strong form efficient price. Afterwards, the market maker observes the posted prices of asset b and of the ETF (or the APs arbitrage opportunity, see Footnote 1).

[1] In this paper, the updating is due to observing the prices in all markets. However, the same results are obtained when APs arbitrage away any price difference between the ETF and the underlying assets.

In an economy without an ETF, the price of asset b does not contain relevant information for pricing asset a because the asset values are independent. But in an economy with an ETF, the market maker pricing asset a must take the price of asset b into account because both assets are part of the ETF. The market maker conditions on the price of asset b when extracting information about the value of asset a from the price of the ETF. This cross-market learning leads to a long-term correlation between the prices of fundamentally independent assets.

In addition, ETFs introduce a new form of cross-market trading. Consider now speculator B who knows the value of asset b. At $t = 1.5$, she observes the posted price of asset a. Without an ETF, speculator B does not gain from knowing the price of asset a. However, in an economy with an ETF, speculator B gains an informational advantage over the market maker pricing asset a because speculator B can distinguish the price components of the ETF. For speculator B, the ETF price reveals only information about the value of asset a. All information used by the market makers is public, such that speculator B gains an informational advantage over the market maker pricing asset a and profitably trades asset a in the second period.

Moreover, speculator B also trades asset a in $t = 1$, although speculator B does not have an informational advantage over the market maker pricing asset a. Knowing the value of asset b allows speculator B to predict the price of the ETF at $t = 1.5$. For example, if the value of asset b is high, speculator B expects the price of the ETF to be high. A high price of the ETF signals a high value of the underlying assets, and market maker increases the price of asset a. Speculator B can then unwind his position in asset a at a profit.

In addition, speculator B trades asset a in the direction of his own signal, which protects her informational advantage over the market makers. In the interim period, market makers use publicly observable prices to adjust their valuation of the assets. If speculator B traded asset a in the first period, she knows that the price of asset a is too high. Knowing that the price of asset a is too high implies an additional informational advantage over the market maker pricing asset b and protects the informational advantage that speculator B has over the market maker pricing asset b. The inference of the market maker pricing asset b is more difficult if the price of both assets is high than if only the price of asset b is high. Such a destabilizing trading behavior is not optimal in the standard Kyle (1985)-model, but the additional learning induced by ETFs leads to non-informational cross-market trading in equilibrium.

The cross-market learning and cross-market trading introduced by the ETF generates a correlation between the prices of fundamentally independent assets. Idiosyncratic shocks are then propagated through the price system of the entire economy. Additionally, the learning and trading behavior in an ETF-economy has a dichotomous effect on price efficiency. Cross-market trading in the first period is non-informational and introduces additional noise into the first-period order flow of the underlying assets, which *reduces short-term price informativeness*. On the contrary, the additional learning *increases long-term price informativeness*. At the same time, the expected profit of an informed speculator is lower in an economy with an ETF than in an economy without an ETF, such that her willingness to pay for a signal about the value of the underlying asset is lowered. The market maker makes zero expected profits in equilibrium, such that the expected profit of the informed speculator equals the expected loss of the noise traders. The expected loss of noise traders is thus lower in an ETF-economy than in an economy without an ETF.

Related literature This paper is mainly related to four strands of literature. First, I contribute to the growing debate among regulators, practitioners and academics about the effect of ETFs on financial markets, as summarized in Ben-David et al. (2017). Related to this paper, Bhattacharya and O'Hara (2018) develop a Kyle (1985)-style model of trading in an ETF and strictly segmented underlying markets. Bhattacharya and O'Hara (2018) assume that asset values follow a factor structure to analyze the propagation of idiosyncratic and systemic shocks through the price system. I assume that traders have access to both underlying asset markets, use purely idiosyncratic asset values, and analyze the effects of dynamic optimization, which allows me to identify a novel kind of cross-market trading.

Malamud (2016) develops a dynamic general equilibrium model showing that the creation / redemption mechanism of ETFs propagates temporary demand shocks across periods, leading to momentum. In the model, trade occurs only for risk-sharing purposes. Bond and Garcia (2022) use a rational expectations model to show that a reduction of the costs of indexing leads to more usage of index instruments which reduces price informativeness (see also Stambaugh 2014). On the contrary, Buss and Sundaresan (2023) show that passive ownership increases price informativeness because firms are incentivized to allocate more capital to risky growth opportunities, which induces investors to acquire more precise information. In contrast to these authors, I am using a pure information channel to show that ETFs increase long-term price informativeness, but reduce short-term price informativeness.

Empirically, it remains an open question whether ETFs reduce or increase price informativeness. Hamm (2014) finds that ETF ownership is positively related to a stock's illiquidity. Ben-David et al. (2018) and Krause et al. (2014) show that APs arbitrage activity increases the volatility of the underlying stocks, in line with D. C. Brown et al. (2021). Da and Shive (2018) and Agarwal et al. (2018) document that a higher ETF ownership contributes to return comovement at the fund and stock level and Todorov (2019) finds a significant non-fundamental price component in VIX futures due to ETF rebalancing. Positive effects of ETFs on price discovery due to the transmission of fundamental information to less liquid stocks are found by Glosten et al. (2021) and Hasbrouck (2003). Overall, these studies show that ETFs transmit non-fundamental shocks through the pricing system. This paper provides a theoretical framework that suggests that the information effects of ETFs depend on the time-horizon.

Second, the paper is related to the literature on cross-market trading. Pasquariello (2007) demonstrates that speculators trade strategically across assets to mask their informational advantage in one asset whenever asset values are correlated (non-diagonal variance-covariance matrix). The same intuition holds in my model: Essentially, being a redundant asset, the

ETF transform a diagonal variance-covariance matrix of all asset values in the economy into a non-diagonal variance-covariance matrix. Bernhardt and Taub (2008) show that prices are more correlated than asset fundamentals if speculators combine private information about multiple assets with information contained in prices due to cross-market learning (Baruch et al. 2007). Moreover, Cespa and Foucault (2014), using a rational expectations framework, find that liquidity shocks in one asset spill over to other assets due to learning about the value of one asset from prices of other assets.[2] In this paper, I show that these effects emerge naturally if an ETF–and more generally an index security—is introduced into an economy.

Third, this article is related to older microstructure models that analyze the impact of an index on optimal trading. Subrahmanyam (1991) examines the optimal strategy for discretionary liquidity traders that can trade the underlying assets or an index and shows that the adverse selection costs are lower in the index. Gorton and Pennacchi (1993) show that an index can improve the welfare of uninformed traders. The focus of my paper is different from these papers, as I examine inter-market information transmission and do not investigate welfare implications.

Fourth, my paper is also related to the literature on manipulative or excessive trading. Brunnermeier (2005) shows that speculators trade partially with the intention of increasing their informational advantage if they know a noisy version of a public signal that will arrive later. A similar structure is present in my model, where the price of the ETF in the interim period can be thought of as a public signal. Moreover, Chakraborty and Yılmaz (2004) show that manipulative trading can emerge in a Kyle (1985)-equilibrium when there is uncertainty about the existence of an informed speculator. Fishman and Hagerty (1995), John and Narayanan (1997), and Huddart et al. (2001) demonstrate that manipulative trading may occur due to the existence of public disclosure rules.

The paper is organized as follows. In Section 1, I present the model and the benchmark

[2]Other authors have studied strategic trading and multiple assets (Boulatov et al. 2013, Caballe and Krishnan 1994), while Cespa and Foucault (2014) and Malamud and Rostek (2017) show under which conditions market fragmentation can arise in equilibrium.

equilibrium without an ETF as in Kyle (1985). I show that there is no cross-market learning, and therefore also no cross-market trading, without an ETF. In Section 2, I present the model with an ETF and study the equilibrium obtained in this market. In Section 3, I compare the trading behavior across these two settings and analyze patterns of liquidity and price informativeness, and Section 4 concludes. Most proofs are delegated to the Appendix.

1 Equilibrium without an ETF

Consider a two-period model of Kyle (1985)-style trading in two independent markets. In each market $m \in \{a, b\}$, a single risky asset with liquidation value $v_m \sim \mathcal{N}(0, \tau_v^{-1})$ is traded.[3] The joint distribution of fundamental values of the risky assets is

$$\mathbf{V} = \begin{pmatrix} v_a \\ v_b \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tau_v^{-1}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right], \tag{1.1}$$

where I assume that the fundamental values are independent.

Market Participants. In each asset market, there are three types of agents: One informed speculator, one competitive market maker, and a mass of liquidity traders. Before trading starts, each speculator observes the fundamental value v_m, $m \in \{a, b\}$ of one asset but not of the other asset. Speculator A (B) knows the value of asset a (b). Liquidity traders buy or sell for exogenous reasons. In each asset market $m \in \{a, b\}$ and in each period $t \in \{1, 2\}$, they place a random net order of $z_{m,t}$, with $z_{m,t} \sim \mathcal{N}(0, \tau_z^{-1})$. The demand of liquidity traders is independent of all other random variables in the model and independent across markets and time periods. For simplicity, I assume that the variance of liquidity orders is identical in both markets and periods.

In each market, a single risk-neutral market maker observes the aggregate order flow (but not the order flow in the other market) and sets the price. As in Kyle (1985), unmodeled

[3]The assumption that the liquidation value is distributed around 0 is for simplicity only and without loss of generality.

competition drives the market maker's expected profits to zero, such that he is setting semi-strong form informationally efficient prices.

Timing of Trade There are two trading rounds and an interim period, $t \in \{1, 1.5, 2\}$. In period 1, informed speculators trade according to their information. The price set in period 1 reflects the information contained in the order flow in the respective market. Market makers cannot observe the contemporaneous order flow in other markets. After the first round of trading, at $t = 1.5$, the market makers observe the prices set in the other markets and update their valuation.[4] In period 2, speculators again trade according to their information, and market makers set prices after observing the order flow in their respective market. After period 2, the asset is liquidated and the price is equal to v_m.

As in all Kyle (1985)-type models, the market makers' information sets include the past order flow, and speculators submit market orders taking into account the price impact of their orders. Informed traders only trade to exploit their informational advantage. As the noise-trader order flow is independent across periods and markets, the aggregate order flow in each market is given by:

$$q_{m,t} = x^A_{m,t} + x^B_{m,t} + z_{m,t} \tag{1.2}$$

where $x^A_{m,t}$, $x^B_{m,t}$ is the order submitted by speculator A (informed about v_a) or speculator B (informed about v_b) at time t, respectively. The sequence of actions is highlighted in Figure 1.1 and the information structure is summarized in Table 1.1, where $P_{m,t}$ denotes the price of asset m in period t.

The information structure (but not the information) is common knowledge among all market participants, that is, everyone knows that speculator A knows v_a (and everyone knows, that everyone knows, etc.). Market makers observe the aggregate order flow, such they do not know the exact order placed by informed or uninformed traders. In period 1.5,

[4]Note that there is a one-to-one mapping between prices and order flows. Observing price changes is thus equivalent to observing order flows.

Figure 1.1: Timeline of the model without an ETF.

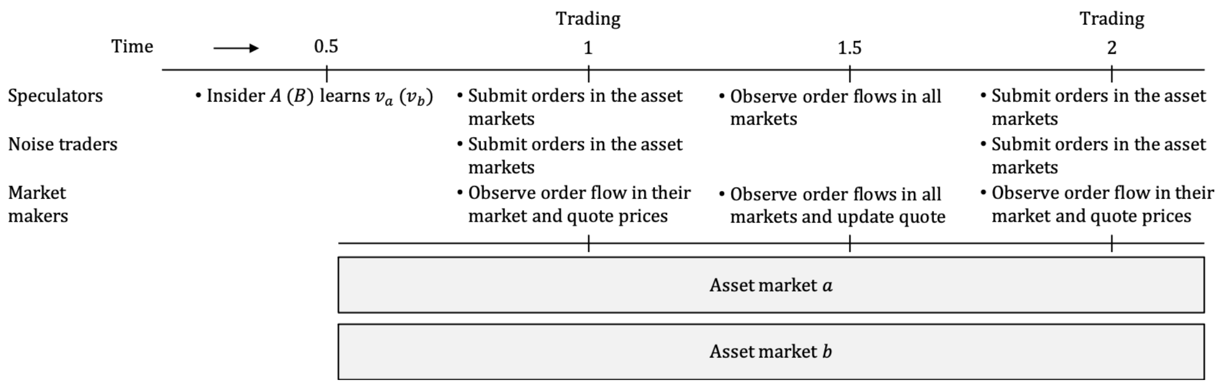


Table 1.1: Information structure

	Period		
Player	$t = 1$	$t = 1.5$	$t = 2$
Market maker m	$q_{m,1}$	$P_{a,1}, P_{b1}$	$P_{a,1}, P_{b1}, q_{m,2}$
Speculator A	v_a	$v_a, P_{a,1}, P_{b1}$	$v_a, P_{a,1}, P_{b1}$
Speculator B	v_b	$v_b, P_{a,1}, P_{b1}$	$v_b, P_{a,1}, P_{b1}$

Table 1.1 summarizes the information structure of the economy, as also described in Figure 1.1.

each trader also observes the price in the other market and can infer the order flow in this market from the prices. The risk-neutral market maker sets the execution price $P_{m,t}$ after observing the aggregate order flow. Setting informationally efficient prices in equilibrium means that $P_{m,1} = \mathbb{E}(v_m|q_{m,1})$ and $P_{m,2} = \mathbb{E}(v_m|P_{a,1}, P_{b,1}, q_{m,2})$. I refer to the information set of player $k \in \{A, B, MM\}$ in period t as $\mathcal{I}_t^k$, such that the information set of both market makers in period 1.5 is $\mathcal{I}_{1.5}^{MM} = \{P_{a,1}, P_{b1}\}$.

A sequentially rational Bayesian Nash Equilibrium of this trading game is given by a strategy profile $\{\{x^{A*}_{m,1}, x^{A*}_{m,2}, x^{B*}_{m,1}, x^{B*}_{m,2}, P^*_{m,1}, P^*_{m,2}\}_{m=\{a,b\}}\}$ such that

$$x^{A*}_{m,1} \in \underset{x^A_{m,1}}{\operatorname{argmax}}\, \mathbb{E}(x^A_{m,1}(v_m - P_{m,1})|\mathcal{I}^A_1) \;\; \forall m \in \{a, b\},$$

$$x^{A*}_{m,2} \in \underset{x^A_{m,2}}{\operatorname{argmax}}\, \mathbb{E}(x^A_{m,2}(v_m - P_{m,2})|\mathcal{I}^A_2) \;\; \forall m \in \{a, b\},$$

$$x^{B*}_{m,1} \in \underset{x^B_{m,1}}{\operatorname{argmax}}\, \mathbb{E}(x^B_{m,1}(v_m - P_{m,1})|\mathcal{I}^B_1) \;\; \forall m \in \{a, b\},$$

$$x^{B*}_{m,2} \in \underset{x^B_{m,2}}{\operatorname{argmax}}\, \mathbb{E}(x^B_{m,1}(v_m - P_{m,2})|\mathcal{I}^B_2) \;\; \forall m \in \{a, b\},$$

$$\text{and prices } P_{m,1} = \mathbb{E}(v_m|\mathcal{I}^{MM}_1) \text{ and } P_{m,2} = \mathbb{E}(v_m|\mathcal{I}^{MM}_2) \;\; \forall m \in \{a, b\},$$

where the conditional expectations are derived using Bayes' Rule to ensure belief consistency.

1.1 Characterization of the linear equilibrium

Proposition 1 characterizes a sequentially rational Bayesian Equilibrium in linear strategies as in Kyle (1985).

Proposition 1 There exists a unique linear equilibrium in the two-period model given by

$$x^A_{a,t} = \beta^A_{a,t}\,(v_a - P_{a,t-1}) \qquad\qquad x^A_{b,t} = 0$$

$$P_{m,t} = \kappa_{m,t} + \lambda_{m,t}\, q_{m,t}$$

using $P_{a,0} = \mathbb{E}(v_a) = 0$. The constants $\beta^A_{a,t}, \kappa_{a,t}$, and $\lambda_{a,t}$ are the unique solution to the following equation system:

$$\beta^A_{a,2} = \sqrt{\frac{\tau_{v_{a,1}}}{\tau_z}} \qquad \beta^A_{a,1} = \frac{1}{2\lambda_{a,1}} \cdot \frac{1-2\psi_a\lambda_{a,1}}{1-\psi_a\lambda_{a,1}} \qquad \psi_a = \frac{1}{4\lambda_{a,2}}$$

$$\kappa_{a,2} = P_{a,1} \qquad \kappa_{a,1} = 0$$

$$\lambda_{a,2} = \frac{1}{2}\sqrt{\frac{\tau_z}{\tau_{v_{a,1}}}} \qquad \lambda_{a,1} = \frac{\beta^A_{a,1}\tau_z}{\beta^{A2}_{a,1}\tau_z + \tau_v}$$

$$\tau_{v_{a,1}} = \tau_v + \beta^{A2}_{a,1}\tau_z$$

and the constraints $\lambda_{a,1}(1-\psi_a\lambda_{a,1}) > 0$, $\lambda_{a,2} > 0$. $\beta^B_{b,t}, \kappa_{b,t}, \lambda_{b,t}$ follow by symmetry.

The proof is standard and can be found in Kyle (1985), Admati and Pfleiderer (1988) or Vives (2010). The demand of each speculator is the product of her trading intensity ($\beta^A_{a,t}$) and her informational advantage over the market maker. Moreover, linear strategies retain the normality of all random variables and generate a linear pricing function. As in Kyle (1985), $\lambda_{m,t}$ reflects the price impact of an increase in the market order by one unit and restricts the informed trader's optimal trading quantities. Kyle (1985) interprets $\lambda^{-1}_{m,t}$ as a measure of the market's depth. If $\lambda_{m,t}$ is very small, an increase in the trader's demand has only a small impact upon the market price and the market is perceived as liquid.

1.2 Learning and information properties

The optimal market order size in Proposition 1 depends on the informational advantage of the speculator over the market maker. The informational advantage, in turn, is determined by the speculator's information about the fundamental value of the stock and her trading aggressiveness. While the speculator perfectly knows the liquidation value of the asset, the market maker relies on the information contained in the current and past order flow. The market maker can infer the order flow that occurred at $t = 1$ in the other market $b \neq a$ from $t = 1.5$ onward. However, the order flow in market b (a) is not informative for the value of asset a (b) because the asset values and the noise trader demand are independent across markets.[5] Therefore, there is no *cross-market learning*, defined as learning from the order flow in another market, as highlighted in Corollary 1, which follows from $x^B_{a,t} = x^A_{b,t} = 0$ in Proposition 1.

Corollary 1 Without an ETF, the trading activity in a given market is independent of the trading activity in another market and no market participant can infer additional information from the order flow in another market.

I define trading with the sole intention of increasing one's informational advantage as *manipulative trading*. Manipulative trading is ruled out in the equilibrium given in Proposition 1 because the second-order conditions of the unique linear equilibrium "rule out a situation in which the insider can make unbounded profits by first destabilizing prices with unprofitable trades made at the nth auction, then recouping the losses and much more with trades at future auctions" (p. 1323 Kyle 1985).The second-order condition $\lambda_{a,1}(1 - \psi_a \lambda_{a,1}) > 0$ places an upper bound on $\lambda_{a,1}$, as $1 > \psi_a \lambda_{a,1}$.[6] The upper bound determines the ability of the speculator to trade profitably in $t = 2$, as the value function increases in ψ_a. If the speculator could cheaply manipulate prices today by placing a small, misleading order (large $\lambda_{a,1}$), her expected profit in the next period is low due to the second-order condition. The same condition rules out manipulative trading on the other asset market, as summarized by Corollary 2. This is a direct consequence of Lemma 1 and the second-order conditions.

Corollary 2 Without an ETF, informed traders do not engage in manipulative trading.

[5] If the liquidation values of the assets were correlated, a market maker could gain additional information from the order flow in other markets. This case is, for example, analyzed by Boulatov et al. (2013) and Cespa and Colla (2020). If the demand of noise traders were correlated, observing the order flow in another market would provide the market maker with additional information about the noise trader demand in her own market. Prices could then be updated.

[6] The second-order condition does also impose a lower bound of $\lambda_{a,1} > 0$.

2 Equilibrium with an ETF

Let us now introduce an ETF whose value is the weighted average value of the underlying assets into the economy analyzed in the previous Section. The value of the ETF is given by $v_e = \omega_a v_a + \omega_b v_b$ with $\omega_a + \omega_b = 1$. Therefore, the joint distribution of the fundamental values of the economy is

$$\mathbf{V}_e = \begin{pmatrix} v_a \\ v_b \\ v_e \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tau_v^{-1} \begin{pmatrix} 1 & 0 & \omega_a \\ 0 & 1 & \omega_b \\ \omega_a & \omega_b & \omega_a^2 + \omega_b^2 \end{pmatrix}\right]. \tag{2.1}$$

Introducing an ETF leads to a non-diagonal variance-covariance matrix, leading to fundamentally connected assets Pasquariello (2007). Fundamentally connected assets allow market participants to learn from the prices of other assets. To separate the effects of the additional learning channel from the effects of strategic trading in correlated securities, I assume that the ETF and the underlying assets trade in segmented markets. Participants are exogenously assigned to trade either the ETF or the assets and cannot trade the other asset class. Implicitly, I assume that only APs can trade both the asset and the underlying securities which leads to an updating of the prices of all assets in the interim period $t = 1.5$.

Assumption 1 (Segmented markets) The ETF and the underlying assets trade in segmented markets. Traders are exogenously assigned to trade either the ETF or the underlying assets, but cannot access the other asset class.

Assumption 1 is mainly for tractability and relaxing it does not change the results qualitatively (see Theorem 1 in Boulatov et al. 2013). If asset market speculators could trade the ETF, their optimization problem would need to balance the information revelation due to trading the underlying assets and the ETF. I could no longer separate the information effects due to the introduction of the ETF from the alternation of the speculator's optimization problem.

Moreover, I focus on an equally-weighted ETF as this allows me to focus on symmetric equilibria. If the assets were not equally weighted, the ETF order flow would be more informative about the overweighted asset, and the updating problem would be asymmetric. Assuming an equally-weighted ETF does not affect the results qualitatively, but allows us to simplify the notation considerably.

Assumption 2 (Equally-weighted ETF) I assume an equally-weighted ETF ($\omega_a = \omega_b = 0.5$).

Market Participants. The ETF is traded by *two* risk-neutral informed speculators and a unit mass of noise traders; and priced by a dedicated market maker. Before trading, each speculator observes the liquidation value of one underlying asset, but not about the other. Therefore, one speculator is informed about v_a and the other speculator is informed about v_b.[7] Liquidity traders trade for exogenous reasons and their net demand is $z_{e,t} \sim \mathcal{N}(0, \tau_{z,e}^{-1})$. I assume that $\tau_{z,e} = \eta^{-2}\tau_z$, $\eta > 1$ to model the fact that ETFs are more liquid than underlying assets. The risk-neutral market maker observes the aggregate order flow in the ETF, but not in the underlying asset markets, and sets semi-strong form informationally efficient prices due to unmodeled competition.

Timing of Trade. The timeline is essentially the same as in the previous section, with the addition of the ETF market. Figure 2.1 summarizes the sequence of actions, and the information structure is summarized in Table 2.1.

The information structure is common knowledge among all market participants. The information set of an informed trader does not change from $t = 1.5$ to $t = 2$, and I refer to both information sets for asset speculator A as $\mathcal{I}_2^A$ (and analogously for all other informed traders). The definition of a sequentially rational Bayesian Nash Equilibrium remains the

[7]This assumption ensures that trading in the ETF replicates the information pattern that would emerge if the asset market traders could trade the ETF.

Figure 2.1: Timeline of the model with an ETF present in the economy.

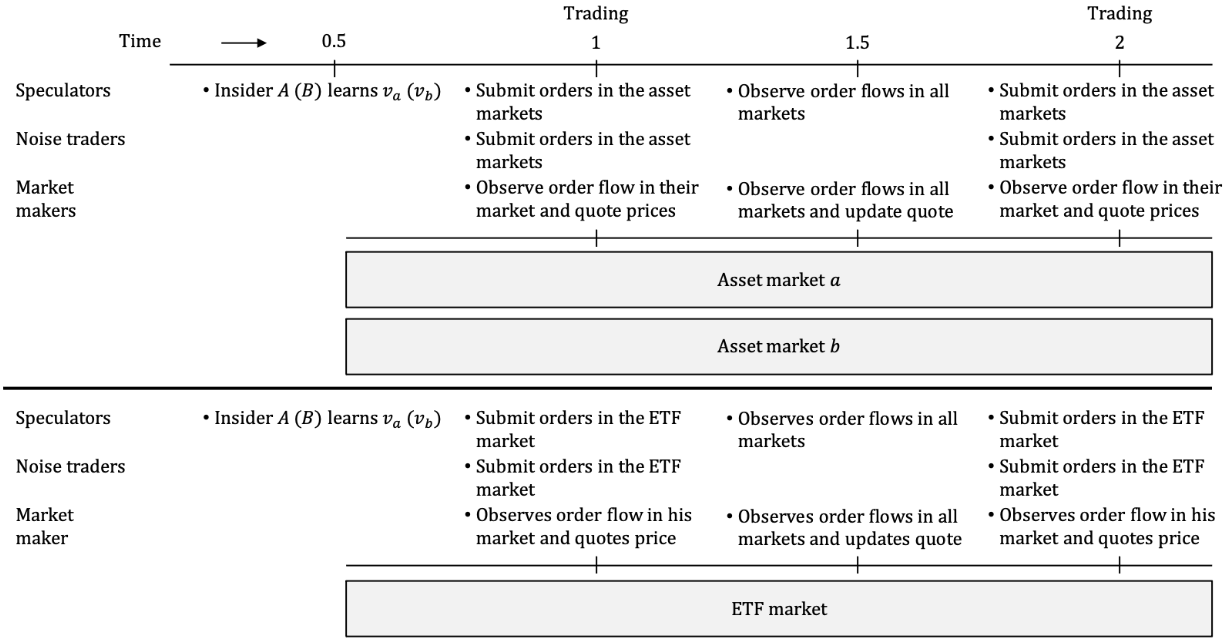


Table 2.1: Information structure

	Period		
Player	$t = 1$	$t = 1.5$	$t = 2$
Asset market maker	$q_{m,1}$	$P_{a,1}$, $P_{b,1}$, $P_{e,1}$	$P_{a,1}$, $P_{b,1}$, $P_{e,1}$ q_{m2}
Asset speculator A	v_a	v_a, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$	v_a, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$
Asset speculator B	v_b	v_b, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$	v_b, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$
ETF market maker	$q_{e,1}$	$P_{a,1}$, $P_{b,1}$, $P_{e,1}$	$P_{a,1}$, $P_{b,1}$, $P_{e,1}$ $q_{e,2}$
ETF speculator A	v_a	v_a, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$	v_a, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$
ETF speculator B	v_b	v_b, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$	v_b, $P_{a,1}$, $P_{b,1}$, $P_{e,1}$

Table 2.1 summarizes the information structure of the economy with an ETF, as also described in Figure 2.1.

same as before, except for adding optimality conditions for trading in the ETF.

2.1 Characterization of the linear equilibrium

In this section, I describe the linear equilibrium that arises in a market with an ETF on top of two uncorrelated assets. In contrast to the model discussed in Section 1, the order flow in the ETF provides an additional source of information about the value of the underlying assets. This leads to a coupling of asset markets and makes cross-market learning and trading profitable. Speculators have an incentive to engage in *manipulative trading*, i.e., trading with the sole purpose of increasing their informational advantage (Brunnermeier 2001, Brunnermeier 2005).

Manipulative trading is profitable because market makers learn in stages. In t = 1, market makers set prices to reflect the information in their own market. Afterwards, they can observe the price changes in all other markets. As in the economy without an ETF, the market marker pricing asset a can learn about the value of asset b, v_b, by observing the price change in market b. Contrary to the economy without an ETF, this information is useful for pricing asset a once an ETF is introduced. As asset a is part of the ETF, the price of the ETF contains relevant information about the value of asset a and asset b. The market maker pricing asset a cannot distinguish this information, but he accounts for the value of asset b by taking the price of asset b into account. Thus, markets become coupled after the introduction of an ETF.

This cross-market learning after the first trading round induces asset speculator A to interfere with the signal of the market maker. As the net order flow from liquidity traders is normally distributed, any aggregate order flow from $(-\infty, +\infty)$ can arise in equilibrium. However, this signal jamming trade is not random, as in Huddart et al. (2001), but proportional to the signal of the speculator. Intuitively, knowing the value of asset a provides speculator A with an early signal of the price of the ETF. If the value of asset a is very high, for example, the trader expects that the price of the ETF will also be high. Thus, she knows that the market maker pricing asset b will receive a high signal at the interim period and will therefore increase the price of asset b in the interim period. Speculator A is trying to profit from this expected short-term price increase by buying asset b in the first period. This intuition is represented in Proposition 2

Proposition 2 There exists a symmetric linear equilibrium in the two-period model. The optimal orders and pricing function in the first period are given by

$$x^A_{a,1} = \beta^A_{a,1} v_a \qquad x^A_{b,1} = \gamma^A_{b,1} v_a$$

$$x^A_{e,1} = \beta^A_{e,1} v_a$$

$$P_{a,1} = \lambda_{a,1} q_{a,1} \qquad P_{e,1} = \lambda_{e,1} q_{e,1}$$

After the first period, at $t = 1.5$, each market maker updates his expectation of v_m from $P_{m,1}$ to $\mu_{m,p}$. The optimal orders in the second period are therewith given by:

$$x^A_{a,2} = \beta^A_{a,2}(v_a - \mu_{ap}) \qquad x^A_{b,2} = \gamma^A_{b,2}(\mathbb{E}(v_b|\mathcal{I}^A_2) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\mu_{b,p})$$

$$x^A_{e,2} = \beta_{e,2}(\omega_a v_a - \mu_{e,p}) + \omega_b \gamma_{e,2} \mathbb{E}(v_b|\mathcal{I}^A_2)$$

$$P_{a,2} = \kappa_{a,2} + \lambda_{a,2} q_{a,2} \qquad P_{e,2} = \kappa_{e,2} + \lambda_{e,2} q_{e,2}$$

The coefficients and second-order conditions for all markets are the (numerical) solutions to the equation system given in Appendix B. The equations for market b and the trader informed about v_b in both, the asset market and the ETF, follow by symmetry. Note that $\tau_{v|\mathcal{I}^A_2}$ denotes the conditional precision of the signal of speculator A after observing the $t = 1$ order flows in all markets.

As in Section 1, speculators trade proportional to their informational advantage. The only exception is the trading of speculator A in market b in $t = 1$ (and of B in market a). The optimal order in the alien asset market is independent of the realization value of the respective asset and of an existing informational advantage. Instead, this order is placed to profit from a short-term price movement and to increase the informational advantage in $t = 2$ (manipulative trading).

The trading aggressiveness of speculator A in $t = 2$ in market b is increasing in the precision of her posterior about the value of asset b. Moreover, $\beta^A_{a,2}$ is decreasing in the precision of the posterior of asset speculator B about the value of asset a due to competition effects, but always larger than the trading aggressiveness of speculator B in market a. Also the market maker is accounting for the precision of the posterior of the asset market speculator engaging in cross-market trading when setting prices in $t = 2$.[8] Assumption 2 allows me to focus on symmetric equilibria, while the linearity of the equilibrium retains the normality of all coefficients.

2.2 Learning and information properties

The proof of Proposition 2 makes use of backwards induction. In order to solve the continuation game in $t = 2$, the information structure prior to the trading round has to be derived. To do so, I propose an arbitrary strategy profile $\beta^A_{a,1}, \beta^B_{b,1}, \gamma^A_{b,1}, \gamma^B_{a,1}, \beta^A_{e,1}, \beta^B_{e,1}, \lambda_{a,1}, \lambda_{b,1}, \lambda_{e,1}$ for $t = 1$, which is considered to be an equilibrium. After the first period, in $t = 1.5$, all participants observe the aggregate order flows $q_{a,1}, q_{b,1}, q_{e,1}$ and update their beliefs accordingly. Let us consider asset market speculator A. Knowing v_a and her own demand in the markets a and b, the three order flows provide her with three signals about v_b, given by:

$$s_{b,1} = \frac{q_{a,1} - x^A_{a,1}}{\gamma^B_{a,1}} = v_b + \frac{z_{a,1}}{\gamma^B_{a,1}}$$
$$s_{b,2} = \frac{q_{b,1} - x^A_{b,1}}{\beta^B_{b,1}} = v_b + \frac{z_{b,1}}{\beta^B_{b,1}}$$
$$s_{b,3} = \frac{q_{e,1} - \beta^A_{e,1} v_a}{\beta^B_{e,1}} = v_b + \frac{z_{e,1}}{\beta^B_{e,1}}$$

[8]The result follows from the equation of $\lambda_{a,2}$ in the Appendix. Moreover, note that from equations (B.4) and (B.5), it follows that $x^A_{a,1}$ and $x^A_{b,1}$ are functions of v_a.

With this information, her posterior expectation and variance of v_b are:

$$
\begin{aligned}
Var(v_b|\mathcal{I}_2^A) =& \left(\tau_{v|\mathcal{I}_2^A}\right)^{-1} = \left(\tau_v + (\gamma_{a,1}^{B^2} + \beta_{b,1}^{B^2})\tau_z + \beta_{e,1}^{B^2}\tau_{z,e}\right)^{-1}, \qquad (2.2)\\
\mathbb{E}(v_b|\mathcal{I}_2^A) =& \frac{1}{\tau_{v|\mathcal{I}_2^A}}\Big((\beta_{b,1}^B q_{b,1} + \gamma_{a,1}^B q_{a,1})\tau_z - \\
& (\beta_{b,1}^B\ x_{b,1}^A + \gamma_{a,1}^B x_{a,1}^A)\tau_z + \beta_{a,1}^B\tau_{z,e}(q_{e,1} - x_{e,1}^A)\Big) \qquad (2.3)\\
=& \left(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}\right) v_b + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}\epsilon_{\mathcal{I}_2^A}
\end{aligned}
$$

where $\epsilon_{\mathcal{I}_2^A} \sim \mathcal{N}\left(0, \tau_v^{-2}\ \left[(\gamma_{a,1}^{B^2} + \beta_{b,1}^{B^2})\tau_z + \beta_{e,1}^{B^2}\tau_{z,e}\right]\right)$ denotes the signal error. As can be seen from Equation (2.3), the posterior expectation of v_b given v_a and the order flow in each asset market is a convex combination of v_b and a signal error. The same holds for the posterior expectation of v_a given v_b and the order flows.

Speculator A (B) knows v_a (v_b), such that her filtration is finer than that of the market maker who cannot extract the order flow components due to v_a (or v_b). While the market maker also receives three signals about v_a, the noise is correlated due to v_b. Using the Projection Theorem, I find the market maker's posterior variance and expectation of v_b as

$$
\begin{aligned}
Var(v_b|\mathcal{I}_{1.5}^{MM}) = \left(\tau_{vp}^b\right)^{-1} = \Big(\tau_v + (\gamma_{a,1}^{B^2} + \beta_{b,1}^{B^2})\tau_z + \beta_{e,1}^{B^2}\tau_{z,e}\frac{(\tau_v + (\gamma_{b,1}^{A^2} + \beta_{a,1}^{A^2})\tau_z)}{\beta_{e,1}^{A^2}\tau_{z,e} + (\tau_v + (\gamma_{b,1}^{A^2} + \beta_{a,1}^{A^2})\tau_z)}\\
- (\beta_{a,1}^A\ \gamma_{a,1}^B + \beta_{b,1}^B\ \gamma_{b,1}^A)\tau_z\frac{(\beta_{a,1}^A\ \gamma_{a,1}^B + \beta_{b,1}^B\ \gamma_{b,1}^A)\tau_z + 2\beta_{e,1}^A\ \beta_{e,1}^B\tau_{z,e}}{\beta_{e,1}^{A^2}\tau_{z,e} + (\tau_v + (\gamma_{b,1}^{A^2} + \beta_{a,1}^{A^2})\tau_z)}\Big)^{-1}, \qquad (2.4)\\
\mathbb{E}(v_b|\mathcal{I}_{1.5}^{MM}) = \mu_{bp} = \left(\tau_{vp}^b\right)^{-1}\ \left[(\beta_{b,1}^B q_{b,1} + \gamma_{a,1}^B q_{a,1})\tau_z - (\beta_{a,1}^A q_{a,1} + \gamma_{b,1}^A q_{b,1})\ \tau_z\ K_b + q_{e,1}\tau_{z,e}K_{e,b}\right] \qquad (2.5)
\end{aligned}
$$

with

$$
\begin{aligned}
K_b &= \frac{(\beta_{a,1}^A\ \gamma_{a,1}^B + \beta_{b,1}^B\ \gamma_{b,1}^A)\tau_z + \beta_{e,1}^A\beta_{e,1}^B\tau_{z,e}}{\beta_{e,1}^{A^2}\tau_{z,e} + (\tau_v + (\gamma_{b,1}^{A^2} + \beta_{a,1}^{A^2})\tau_z)}, \text{ and}\\
K_{e,b} &= \frac{(\beta_{e,1}^B(\tau_v + (\beta_{a,1}^{A^2} + \gamma_{b,1}^{A^2})\tau_z) - \beta_{e,1}^A(\beta_{a,1}^A\ \gamma_{a,1}^B + \beta_{b,1}^B\ \gamma_{b,1}^A)\tau_z}{\beta_{e,1}^{A^2}\tau_{z,e} + (\tau_v + (\gamma_{b,1}^{A^2} + \beta_{a,1}^{A^2})\tau_z)}.
\end{aligned}
$$

Comparing Equation (2.2) with Equation (2.4) we see that the market maker's updating leads to a less precise posterior distribution of v_b than the updating of speculator A. Indeed, while the first two summands in both equations are identical, the third summand in Equation (2.4) is the same as in Equation (2.2) discounted by the factor $\frac{\tau_v+(\gamma_{b,1}^{A^2}+\beta_{a,1}^{A^2})\tau_z}{\beta_{e,1}^{A^2}\tau_{z,e}+(\tau_v+(\gamma_{b,1}^{A^2}+\beta_{a,1}^{A^2})\tau_z)} < 1$. Moreover, the last term in Equation (2.4) accounts for the correlated noise in the signals of the market maker, which reduces the posterior precision. Overall, this leads to $\tau_{vp}^b < \tau_{v|\mathcal{I}_2^A}$. The same intuition holds for the posterior expectation, where the first summand is the same as in Equations (2.3) and (2.5).

This analysis shows that cross-market learning occurs in equilibrium (Lemma 3) if an ETF is present in the economy. The ETF links the underlying, fundamentally uncorrelated assets and makes the economy fundamentally correlated and the trading activity of fundamentally unrelated assets is correlated if an ETF is introduced into the economy.

Corollary 3 In a market with an ETF representing uncorrelated underlying assets, cross-market learning occurs in equilibrium. This leads to a linkage in the trading activity across markets.

Proof. The statement follows from Step 1 of the proof of Proposition 2. □

The difference between the conditional expectation of the market maker and of the speculator is exploited by the speculator in the second period when trading in the alien asset market and in her own market. In $t = 2$, all traders face a static Kyle (1985)-trading game with imperfect competition. Each trader wants to trade aggressively to exploit the informational advantage, but must balance the order's price impact, leading to an interior equilibrium.

Using backwards induction, one has to check for optimal strategies in $t = 1$ after deriving optimal continuation strategies in $t = 2$. Trading in $t = 1$ does not only affect the profits in $t = 1$ but does also alter prices $P_{a,1}, P_{b,1}$ or $P_{e,1}$ and the conditional expectations of other market participants. An equilibrium is reached if no trader wants to deviate from the proposed strategy profile in $t = 1$.

The value function of each speculator is quadratic in her informational advantage, such that each speculator takes the effect of her trading decision onto her informational advantage into account when submitting orders in $t = 1$. For example, adding noise to the order flow in market b in period $t = 1$ by trading in market b increases the expected informational advantage of speculator A over the market maker in $t = 2$. Moreover, as discussed above, knowing the value of asset a allows her to forecast the price change of all assets at the interim period $t = 1.5$. Therefore, speculators trade in the alien market already in the first period, despite having no informational advantage. Trading in the other market does (a) protect the long-term informational advantage and (b) allows the speculator to profit from short-term price fluctuations. The behavior resembles signal jamming (Brunnermeier 2005, Fudenberg and Tirole 1986, Huddart et al. 2001).

Lemma 1 In a market with an ETF on top of underlying assets, the informed speculators engage in signal jamming / manipulative trading.

3 Comparing equilibria with and without an ETF

In this section, we compare the equilibrium of the asset market with and without the ETF.

3.1 Correlation of asset prices

In the market without an ETF (Section 1), two uncorrelated assets trade in separated markets, and there is neither cross-market learning nor cross-market trading. Introducing an ETF on top of the underlying assets provides an additional source of learning that induces all participants to infer information from the prices of uncorrelated assets. Traders and market makers perform a technical analysis (D. P. Brown and Jennings 1989) of all prices. Cross-market learning occurs after the orders in $t = 1$ have been submitted and price changes are observable for all participants. The prices of fundamentally unrelated assets are correlated in $t = 2$, as shown in Proposition 3.[9]

Proposition 3 In an economy without an ETF and two assets with uncorrelated fundamental values, the second-period prices of the assets $(P_{a,2}, P_{b,2})$ are uncorrelated, i.e.

$$Cov(P_{a,2}, P_{b,2}) = 0 \tag{3.1}$$

while the second-period prices are correlated if an ETF is introduced into the economy:

$$Cov(P_{a,2}^{ETF}, P_{b,2}^{ETF}) \neq 0 \tag{3.2}$$

After $t = 1.5$, speculators informed about the value of one asset, say v_a, have an informational advantage over the market maker regarding the value of the other asset b. Consider asset speculator A who knows v_a and accounts for this information when analyzing the price history of the ETF. The ETF order flow is informative about asset b for asset speculator A, as is the order flow in asset market b. Asset speculator A gains an informational advantage over the market maker, who cannot distinguish the ETF's order flow components due to asset a and b.

In $t = 1$, asset speculator A has the opportunity to increase her informational advantage in $t = 2$ by trading asset b. Neither the market maker nor asset speculator B can infer her order quantity from the aggregate order flow as the net order by liquidity traders is normally distributed. But placing an order $x_{b,1}^A$ affects the price of asset b, which creates an informational advantage over the market maker in market b. The pricing function in $t = 1$ is $P_{b,1} = \lambda_{b,1}\, q_{b,1} = \lambda_{b,1}\, (x_{b,1}^A + x_{b,1}^B + z_{b,1}) = \tilde{P}_{b,1} + \lambda_{b,1}\, x_{b,1}^A$, where $\tilde{P}_{b,1}$ denotes the price that would occur if asset trader A did not trade in market b. Semi-strong form informationally efficient prices ensure that the conditional expectation of the market maker regarding v_b is given by $P_{b,1} = \tilde{P}_{b,1} + \lambda_{b,1}\, x_{b,1}^A$, while the expectation of asset speculator A is $\tilde{P}_{b,1}$. It follows

[9]In this section, I use the symmetry of the equilibrium derived in Proposition 2 and denote $\beta_{a,1}^A = \beta_{b,1}^B = \beta_1, \gamma_{a,1}^B = \gamma_{b,1}^A = \gamma_1, \beta_{e1}^A = \omega_a \beta_e, \beta_{e1}^B = \omega_b \beta_e$.

that $var(v_b|P_{b,1}) > var(v_b|\tilde{P}_{b,1})$, such that asset speculator A gains a more precise signal of v_b from the aggregate order flow in market b than the market maker. This informational advantage is exploited by the speculator in t = 2.

An implication of signal jamming trade in $t = 1$ is that the prices of the underlying assets are correlated in $t = 1$, even before there is an opportunity for cross-market learning. Proposition 4 summarizes the result and shows that prices are uncorrelated in $t = 1$ without an ETF.

Proposition 4 In an economy with two fundamentally uncorrelated assets, the prices of the assets are uncorrelated in the first period. It is

$$cov(P_{a,1}, P_{b,1}) = 0. \tag{3.3}$$

If an ETF is introduced into this economy, the prices are correlated in the first period due to signal jamming trades

$$cov(P_{a,1}^{ETF}, P_{b,1}^{ETF}) = \lambda_{a,1}\,\lambda_{b,1}\,(\beta_{a,1}^{A}\,\gamma_{b,1}^{A} + \gamma_{a,1}^{B}\,\beta_{b,1}^{B})\tau_v^{-1} > 0. \tag{3.4}$$

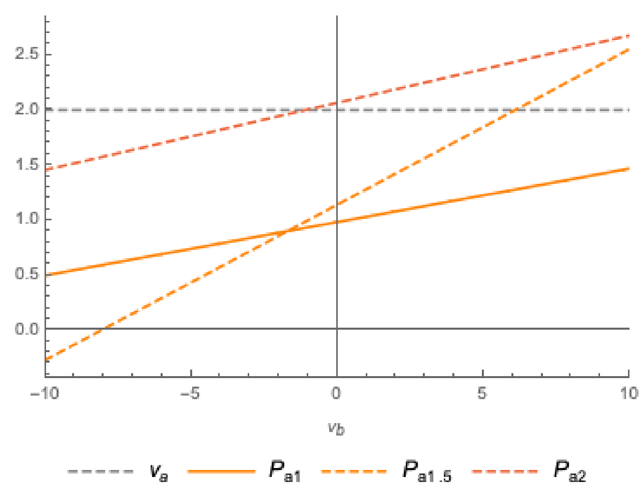
As a consequence of Proposition 3 and 4, an idiosyncratic shock to the fundamental value of asset b affects the price of asset a if an ETF is present in the economy, but not without the ETF. The cross-market learning and cross-market trading in an economy with an ETF propagate a shock to the value of one asset through the entire price system. ETFs introduce a source of market instability that is not present without an ETF (Bhattacharya and O'Hara 2018), as Corollary 4 highlights.

Corollary 4 Let m^c be the complementary element to m in $M = \{a, b\}$. A change of the idiosyncratic value of an asset $m \in \{a, b\}$ does not affect the price of asset m^c in an economy without an ETF. In contrast, an idiosyncratic shock to the value of asset $m \in \{a, b\}$ affects the price of asset m^c in all periods in an economy with an ETF.

The markets for fundamentally uncorrelated assets become coupled due to the introduction of an ETF throughout all time periods. It is a novel prediction of the framework employed here that unrelated asset prices are correlated before cross-market learning occurs in $t = 1$. Bhattacharya and O'Hara (2018) show that the updating of the market maker at period $t = 1.5$ leads to a coupling of the updated asset prices. Two differences to their findings should be highlighted here. First, the assets in the model of Bhattacharya and O'Hara (2018) follow a factor structure with a systematic and idiosyncratic component, making the fundamental value of all assets in the economy. In contrast, the assets considered here are *fundamentally uncorrelated*. Second, Bhattacharya and O'Hara (2018) focus on 'hard-to-access' markets, which prevents cross-market trading.

The effect of a change in the value of asset b on the price of asset a over different periods of time is also visible in Figure 3.1. I plot the price of asset a as a function of v_b. A change in v_b has the least impact on the price of asset a in $t = 1$ and the highest effect on the price in $t = 1.5$.

Figure 3.1: Price of asset a as a function of the value of asset b



It is $v_a = 2, \tau_v = 1, \tau_z = 1, \eta = 2, \omega_a = \omega_b = 0.5$ and all net order flows from noise traders are set to 0.5.

3.2 Trading behavior

The correlation of prices discussed in the previous section allows informed speculators to gain an informational advantage. In period $t = 2$, speculators can infer more information from prices than market makers by conducting a more precise technical analysis. Moreover, manipulative trading in $t = 1$ increases the informational advantage of informed speculators.

Analyzing manipulative trading in isolation, without taking the effect on the posterior of the market makers into account, the expected profit is negative, as is well known in Kyle (1985)-models. The market maker recoups losses expected from trading with informed speculators from uninformed traders. Asset speculator A is uninformed about asset b, such that the expected profit of the position in isolation is negative. Moreover, asset speculator A expects to partially unwind the manipulative position accumulated in $t = 1$ in $t = 2$.

Proposition 5 Asset trader A expects to unwind

$$-\gamma_{b,2}^{A}(1+\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}})\frac{\tau_z}{\tau_{vp}^{b}}\left(2\beta_1\gamma_1-(\beta_1^2+\gamma_1^2)K_b+\omega_a\beta_{e1}\eta^{-2}K_{eb}\right)\, v_a$$

of her t = 1 trade in market b if

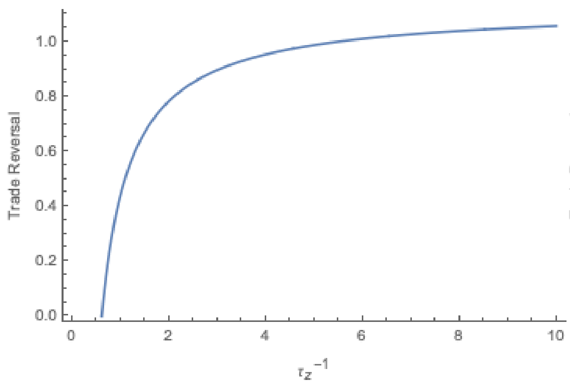
$$2\beta_1\gamma_1(1-K_b)+\omega_a\beta_{e1}\eta^{-2}K_{eb}>(\beta_1-\gamma_1)^2K_b$$

Note that asset speculator A trades in the direction of v_a in market b in the first period, which increases not only $P_{m,1}$ but also the market maker's expectation of the asset value in t = 1.5 given by $\mu_{m,p}$.[10] Hence, the market maker overestimates the value of asset b at the interim period, on average. As asset speculator A has no further informational advantage over the market maker regarding the liquidation value of asset b in t = 1, she expects to profit from this short-term price movement by reverting her position in $t = 2$, as long as

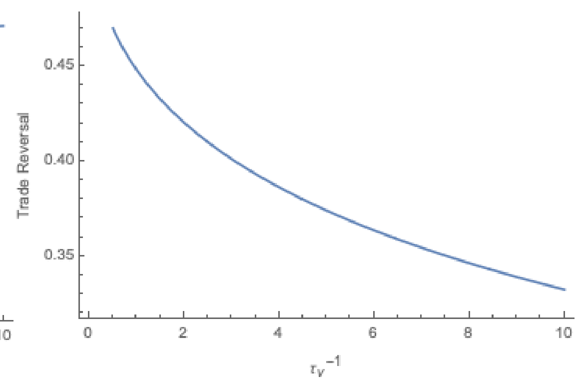
[10]Technically, the reason for this is that K_a given in the Proof of Proposition 2 is smaller than 1, such that $(\beta_1 - \gamma_1 K_a) > 0$. Accordingly, a higher order flow in market b leads to a higher estimate of v_b by the market maker.

there is sufficient noise trading (see Figure 3.2a). The fraction of the manipulative trading position that she expects to unwind is declining with the variance of the fundamental asset value. Intuitively, if the variance of the fundamental values is high, it becomes more likely that the market maker will update the price of asset b in the interim period in the opposite direction of A's position.

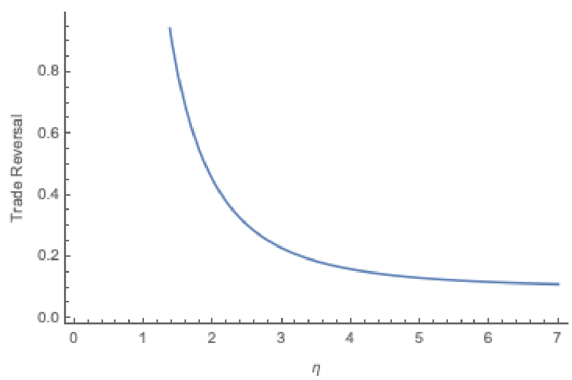
Figure 3.2: Reversal of manipulative trading position



(a) Noise trading variance



(b) Fundamental value variance



(c) ETF market liquidity

Fraction of trade reversal in market $j \neq i$ for different levels of the exogenous parameters. If a parameter is not varied along the x-axis, I set $\tau_v = 1, \tau_z = 1, \eta = 2$. In all panels, it is $\omega_a = \omega_b = 0.5$.

Note that a similar pattern would emerge in an economy without an ETF. If asset speculator A entered a position in market b in t = 1, then she would know that the market maker overestimated the value of asset b and therefore partially unwinds her position in $t = 2$. Manipulative trading only occurs off-equilibrium in an economy without an ETF, while it is an equilibrium behavior in an economy with an ETF.

Trading in market b is optimal for asset speculator A in an economy with an ETF because of the cross-market learning. According to Lemma 1, no cross-market learning occurs in an economy without an ETF. Then, the first-order condition of trader A's total profit with respect to $x^A_{b,1}$ is

$$0 = -2\lambda_{b,1}x^A_{b,1} + \mathbb{E}\big(\frac{\partial V^A_a(x^A_{a,1})}{\partial x^A_{b,1}}|v_a\big) + \mathbb{E}\big(\frac{\partial V^A_b(x^A_{b,1})}{\partial x^A_{b,1}}|v_a\big)$$
$$= \lambda_{b,1}\, x^A_{b,1}\Big(-2 + \frac{1}{8\,\lambda_{b,2}}\,\lambda_{b,1}\Big). \tag{3.5}$$

From this first-order condition, it follows that $x^A_{b,1} = 0$ in equilibrium in an economy without an ETF. The value functions of trader A for both markets are quadratic in her informational advantage over the market maker in $t = 2$, given by $(v_a - \tilde{P}_{a,1})$ in market a and by $\lambda_{b,1}x^A_{b,1}$ in market b. If $x^A_{b,1} > 0$, the market maker in market b overestimates the value of asset b and underestimates the value of asset b if $x^A_{b,1} < 0$. This over- or undervaluation is exploited by asset speculator A in $t = 2$ if she traded in market b in $t = 1$. In an economy without an ETF, the value functions of asset speculator A only depend on her order size in the respective market. This is a consequence of Lemma 1, because her informational advantage depends only on her order size in the respective market and not in the other market.

The equilibrium conditions change when an ETF is introduced into the economy. Lemma 3 demonstrates that cross-market learning occurs in equilibrium in an economy with an ETF. The value functions of asset speculator A, therefore, depend on $x^A_{a,1}$ and $x^A_{b,1}$ simultaneously and are

$$V^A_a(x^A_{a,1}, x^A_{b,1}) = \frac{1}{9\lambda_{a,2}} \cdot \left(\frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}}\right)^2 (v_a - \mu_{ap})^2$$
$$V^A_b(x^A_{a,1}, x^A_{b,1}) = \frac{1}{9\lambda_{b,2}} \cdot \left(\frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}}}\right)^2 (\mathbb{E}(v_b|\mathcal{I}^A_2) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\mu_{bp})^2$$

Similar to an economy without an ETF, her first-order condition wrt. $x^A_{b,1}$ is given by:

$$0 = -2\lambda_{b,1}x^A_{b,1} + \mathbb{E}\big(\frac{\partial V^A_a(x^A_{a,1}, x^A_{b,1})}{\partial x^A_{b,1}}|v_a\big) + \mathbb{E}\big(\frac{\partial V^A_b(x^A_{a,1}, x^A_{b,1})}{\partial x^A_{b,1}}|v_a\big)$$

which implies

$$x^A_{b,1} \propto \mathbb{E}\big(\frac{\partial V^A_a(x^A_{a,1}, x^A_{b,1})}{\partial x^A_{b,1}}|v_a\big) + \mathbb{E}\big(\frac{\partial V^A_b(x^A_{a,1}, x^A_{b,1})}{\partial x^A_{b,1}}|v_a\big) \tag{3.6}$$

Equation (3.6) shows that the optimal order of asset speculator A in market b depends on the effect of $x^A_{b,1}$ on the expected continuation profits in both markets a and b. As these value functions are quadratic in the expected informational advantage, I find that speculator A's trading in market b does not increase the expected current profit $(-2\lambda_{b,1}\ x^A_{b,1} < 0)$. Instead, the effect of the order in market b onto the *continuation profits* leads to the manipulative trading.

3.3 Price informativeness

The previous Section showed that the learning inherent in an ETF economy induces asset market speculators to trade in markets without having an informational advantage. The uninformed trade, in turn, allows speculators to increase their informational advantage over the market maker in t = 2. Cross-market learning and trading thereby leads to a coupling of markets, making the underlying asset prices susceptible to unrelated shocks. Therefore, I next analyze whether ETFs impair or improves price informativeness.

Using a numerical comparison of the trading behavior in an economy with and without an ETF, we can establish the equilibrium properties given in Corollary 5, which are represented graphically in Figure 3.3.

Corollary 5 Comparing an economy with an ETF to an economy without an ETF, I find

$$\beta_2^{ETF} \geq \beta_2 > \beta_1^{ETF} > \beta_1$$

$$\lambda_1^{ETF} > \lambda_2^{ETF} > \lambda_1 > \lambda_2$$

$$\Sigma_1^{ETF} > \Sigma_1$$

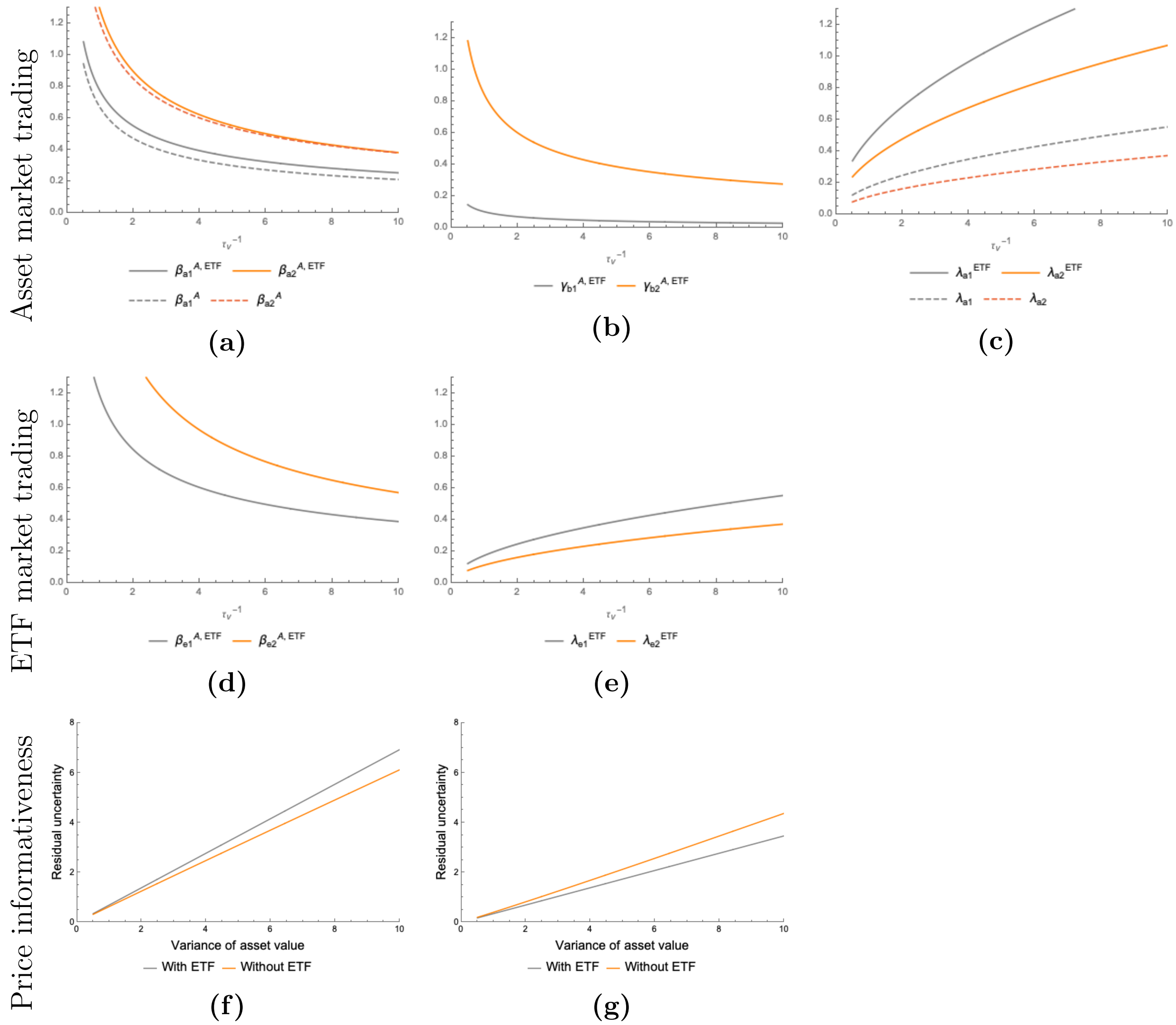
The ranking of the equilibrium coefficients over time represents dynamic optimization as in Kyle (1985). The informed speculator reduces her trading aggressiveness in $t = 1$ to avoid revealing too much information and exploits her retained informational advantage in $t = 2$. The adverse selection problem of the market maker is more severe in the first than in the second trading round. The classical Kyle (1985)-effects are not changed by the presence of an ETF. Moreover, the same effects also hold when trading the ETF (Panel (d) and (e) in Figure 3.3).

An increase in the variance of the fundamental value leads to less aggressive trading and a higher price impact when holding the variance of noise trading constant, as a larger order flow is more likely to be due to fundamental information. Speculators reduce their trading aggressiveness to balance this effect, while the market maker does learn more from the order flow.

Speculators are trading more aggressively when an ETF is present in the economy. In $t = 1$, more aggressive trading results from the presence of another source of noise in the market, as the asset speculator B (A) trades in market a (b). At the same time, the speculator anticipates at t = 1 that the market maker will get an additional signal about v_a from the ETF order flow in t = 1.5. Therefore, the incentive to trade less today to benefit more tomorrow is reduced. In t = 2, the asset speculator A faces competitive pressure in market a from speculator B. This increases her trading aggressiveness relative to the benchmark without an ETF, such that $\beta_2^{ETF} \geq \beta_2$, as in Holden and Subrahmanyam (1992).

Asset markets become less liquid when an ETF is introduced into the economy, as $\lambda_2^{ETF} >$

Figure 3.3: Comparative statics in markets with and without an ETF



Comparative statics between the model with and without an ETF. In Panel (a) and (b), I plot the equilibrium aggressiveness in the asset markets, Panel (c) shows the price impact of the order flow in the asset market, Panel (d) and (e) plot these coefficients for the ETF market, Panel (f) and (g) show the price discovery after t = 1 and t = 2. All coefficients are plotted as a function of the fundamental variance, while I keep $\tau_z = 1, \eta = 2, \omega_a = \omega_b = 0.5$.

λ_2 and $\lambda_1^{ETF} > \lambda_1$. For t = 1, the result is surprising. The presence of asset market trader B in market a introduces an additional source of uncertainty, such that it is expected that the price reacts less to an increased order flow. However, the presence of speculator B in market a as well as the increased learning opportunities of the market maker induce trader A to trade more aggressively. $\lambda_1^{ETF} > \lambda_1$ indicates that the increased trading aggressiveness of trader A dominates the influence of trader B's presence. The conjecture is supported by the low coefficient of $\gamma_{a,1}^{A}$ in Panel (b) of Figure 3.3. In t = 2, the presence of an additional informed trader in the economy with an ETF makes the order flow in the asset market more informative about the fundamental value than in the economy without the ETF. Therefore, the price of an asset reacts more to the order flow in the market, holding the noise trading variance constant.

Overall, cross-market learning and cross-market trading in an economy without an ETF has a dichotomous effect on the price informativeness:

Corollary 6 The introduction of an ETF into the economy worsens short-run price informativeness in the asset market, while it improves long-run price informativeness.

The effect summarized in Corollary 6 is depicted in Panel (f) and (g) of Figure 3.3. In t = 1, irrelevant information mixed with pertinent information affects the prices of underlying assets. The posterior variance in the asset market in an economy with an ETF is

$$\Sigma_{a,1}^{ETF} = \frac{\gamma_1^2 \tau_z + \tau_v}{(\beta_1^2 + \gamma_1^2)\tau_v \tau_z + \tau_v^2},$$

and in an economy without an ETF, the posterior variance after t = 1 is

$$\Sigma_{a,1} = \frac{1}{\beta_1^2 \tau_z + \tau_v}.$$

With an abuse of notation, this implies that

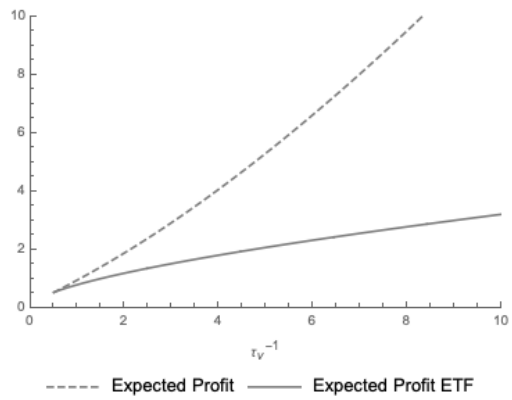
$$\begin{aligned} \Sigma_{a,1}^{ETF} &\geq \Sigma_{a,1} \\ \Leftrightarrow (\gamma_1^2 \tau_z + \tau_v)(\beta_1^2 \tau_z + \tau_v) &\geq (\beta_1^2 + \gamma_1^2)\tau_v \tau_z + \tau_v^2 \\ \Leftrightarrow \qquad (\beta_1 \gamma_1 \tau_z)^2 &\geq 0, \end{aligned}$$

from which we see that $\Sigma_{a,1}^{ETF} \geq \Sigma_{a,1}$. The additional order flow from asset speculator B in market a reduces the price informativeness at the asset level in t = 1 and impairs price discovery. At the same time, the numerical simulations in Panel (f) of Figure 3.3 indicate that the price informativeness in the ETF market is higher than in the underlying asset markets. The fundamental value of the ETF is the weighted average of two uncorrelated assets. Diversification arguments therefore imply that the fundamental value of the ETF is less volatile than that of the underlying assets, which also implies a lower residual uncertainty in the ETF market than in the underlying and more volatile asset markets. In line with Bhattacharya and O'Hara (2018), I find that the introduction of an ETF makes the prices of underlying assets *less* informative. In the long run, however, the additional learning enabled by the ETF reduces the residual uncertainty (see Panel (g) of Figure 3.3). Long-term asset prices in an economy with an ETF are *more* informative than long-term asset prices in an economy without an ETF. To the best of my knowledge, this is a novel prediction of the framework employed here.

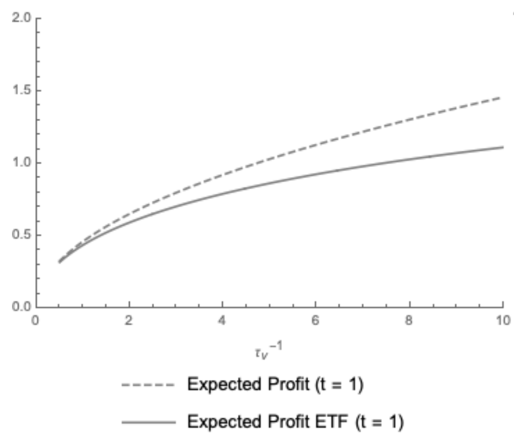
The higher long-run price informativeness in the economy with an ETF in conjunction with the competitive pressure in $t = 2$ leads to a lower expected profit for informed asset market speculators than in an economy without an ETF (see Figure 3.4). Numerical simulations show that the wedge between the expected profit in an economy with and without an ETF is foremost driven by the difference in the expected profit in t = 2.[11] When an ETF is present, the market maker is better informed in t = 2 than when an ETF is not present. Therefore, the expected informational advantage is lower in an economy with an ETF. Moreover, asset speculator A faces competitive pressure from speculator B in t = 2 when an ETF exists, but not without an ETF. This, in turn, does also lower the expected profit of an informed speculator.

[11] Appendix D contains a derivation of the expected profit for both economies.

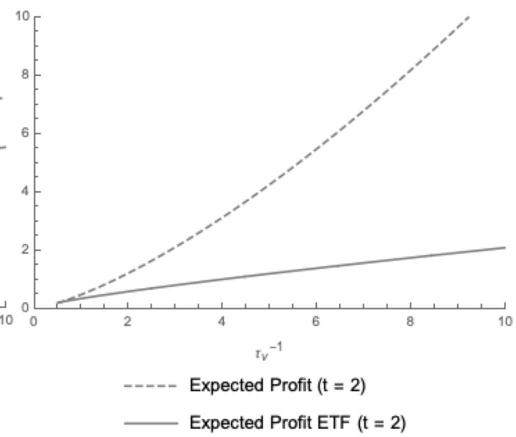
Figure 3.4: Expected profit of informed speculators



(a)



(b)



(c)

Asset trader's expected profit as a function of τ_v^{-1}. The other parameters are $\tau_z = 1, \eta = 2, \omega_a = \omega_b = 0.5$.

4 Concluding remarks

When the first ETF was launched in Toronto 30 years ago, it was only a small notice and a sideshow for specialized investors. Today, ETFs belong to the most prominent investment classes. They are not only a popular financial instrument for private investors searching low-cost index tracking tools, but also used by professional investors seeking factor or asset class exposure without selecting individual securities. This trading activity on the ETF level can lead to an additional price discovery and also to the transmission of unrelated information across the economy.

The growth of assets under management in ETFs has sparked a debate among academics, practitioners, and regulators about the impact of ETFs on the underlying assets. I study the effect of ETFs on underlying assets in a dynamic model with two idiosyncratic securities. These securities are traded in accessible asset markets by an insider knowing the liquidation value of one security but not the other, together with noise traders. Prices are set by competitive market makers. The ETF market is not integrated with the asset markets. Participants are exogenously assigned to trade the ETF or underlying assets. The ETF, in turn, is traded by two informed speculators, whereby each speculator does only know the fundamental value of one of the assets, as well as by liquidity traders. Prices are also set by a competitive market maker.

First, I study a standard setup without an ETF. Although the informed speculators are not restricted to trade only the respective asset market, markets are segmented in this setting. As the asset values are independent of each other, there is no cross-market learning and speculators no cross-market trading.

However, with an ETF, market maker extracts information from the order flow in the ETF market and in the other asset markets. Therefore, prices in one market depend on the trading activity in unrelated asset markets. This leads to a correlation between the prices of fundamentally unrelated assets and a transmission of idiosyncratic shocks through the price system of the economy. Asset markets become interrelated as a result of the introduction of an ETF.

Moreover, the same effect induces speculators to trade across markets, which reinforces the coupling of asset markets. Due to the informational advantage in their own markets, speculators can infer a more precise signal about the value of other assets from the price changes of the ETF, which they exploit profitably by trading across markets in the long run. Additionally, speculators engage in signal jamming trades to profit from short-term price fluctuations due to convergence of prices introduced by the ETF, and to increase their long-term informational advantage over market makers. Introducing an ETF into the economy has dichotomous effects on price informativeness: While short-run prices are less informative, long-term price informativeness is improved.

Throughout the analysis, I impose three potentially restrictive assumptions. First, my analysis focuses on equally-weighted ETFs as this allows me to obtain symmetric equilibria. However, this simplifying assumption does not affect the results qualitatively and could be relaxed to allow for a more general structure of the ETF. In such a setting, the effects are more pronounced for assets that have a higher weight in the ETF basket. If an asset is overweighted in an ETF, market makers and speculators receive more precise information about this asset from the ETF order flow. This differential learning amplifies the identified effects for the overweighted asset and reduces the effects for the underweighted asset.

Second, I assume that the ETF trades in a segmented market. Although the assumption seems restrictive at first, I am focusing on a setting where the information structure pertinent to the underlying asset markets is replicated in the ETF. The ETF is traded by two informed speculators that always have the same information set as the respective speculators in the underlying asset markets. Restricting asset market speculators to trade only in the asset market allows me to separate information effects from effects that are emerging due to an additional trading opportunity. Cespa and Colla (2020) show that trading the same asset in two segmented venues has the similar implications for price discovery. Short-run price discovery worsens, while the long-run price discovery is improved.

Third, I am focusing on pure strategies in my analysis. While this simplifies the analysis and exposition considerably, mixed-strategy equilibria might be found with regard to the first period trading in alien asset markets as they could amplify the benefit of signal jamming trades. However, it is unclear whether there is an equilibrium in mixed strategies. Huddart et al. (2001) and Cespa and Colla (2020) find mixed-strategy equilibria in related models, while Brunnermeier (2005) shows that there are no mixed-strategy equilibria in his setting.

Overall, the article shows that introducing an ETF into an economy may have a substantial effect on the price system, which is not only of academic interest. When asset prices become decoupled from the underlying assets, markets no longer serve as an efficient mean of allocating capital (Goldstein and Guembel 2008). Moreover, I show that ETFs lead to a coupling of asset prices. This might impair the risk-sharing and diversification potential of asset markets. An excessive propagation of shocks through the economy can also lead to excessive volatility of prices, which could destabilize markets. However, the article does not make any normative welfare statements. In order to conduct a welfare analysis, one would need to endogenize the trading activity of liquidity traders, as in Bond and Garcia (2022). Another interesting route could be to analyze the effects of the coupling of asset prices on corporate investment decisions (Schmalz 2018). Finally, it might also be an interesting venue for future research to introduce risk-aversion into the model and study whether ETFs impair the risk-sharing ability of financial markets. Overall, a thorough welfare analysis would allow us to evaluate the effects of introducing an ETF as well as potential policy interventions.

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A Institutional Design of ETFs

ETFs have a trading and pricing mechanism that distinguishes them from other index instruments and from mutual funds. As most economists are familiar with mutual funds, let me highlight the differences between an ETF and an "open-end" mutual fund.

Mutual funds hold a basket of underlying assets. These assets are typically valued at the end of the day by a pricing provider to determine the "net asset value" of the fund. Therefore, in a mutual fund, all transactions occur at the end of the day and at the net asset value.

An ETF also holds a portfolio of assets, but it does interact with the capital markets directly. Instead, the issuer of the ETF (*ETF sponsor*) designates chosen market participants as *Authorized Participants (AP)*, who then interact with capital markets (see Figure A.1). These APs can create or redeem ETF shares by either delivering the constituents of the ETF (in-kind transaction) or by offering the net asset value equivalent of cash to the ETF sponsor (in-cash transaction). In an in-cash transaction, the ETF sponsor buys the replicating basket himself.

When the ETF and the underlying assets are liquid, the APs may "arbitrage" away any price differences between the ETF and the net asset value (NAV), ensuring a low tracking error of the ETF. For instance, if the ETF is trading at a premium to the NAV, the AP would buy the underlying assets in the market and simultaneously short the ETF. At the end of the day, the AP delivers the basket of securities to the ETF sponsor in exchange for ETF shares. Therewith, the AP closes out his position (Ben-David et al. 2017, Bhattacharya and O'Hara 2018).

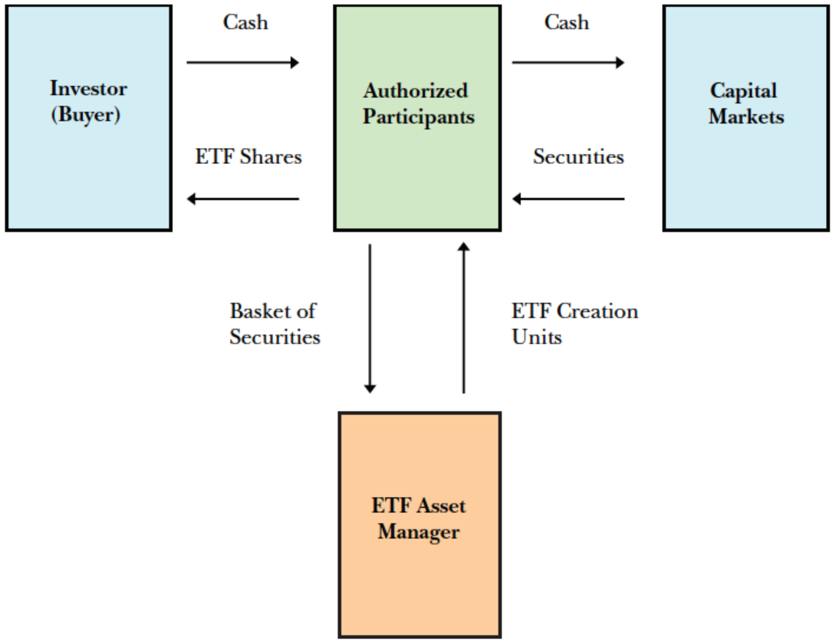

Figure A.1: The ETF architecture (Lettau and Madhavan 2018)

B Equilibrium characterization in an ETF economy

Proposition 2* There exists a symmetric linear equilibrium in the two-period model. The optimal orders and pricing function in the first period are given by

$$x_{a1}^A = \beta_{a1}^A v_a \qquad x_{b1}^A = \gamma_{b1}^A v_a$$

$$x_{e1}^A = \beta_{e1}^A v_a$$

$$P_{a1} = \lambda_{a1} q_{a1} \qquad P_{e1} = \lambda_{e1} q_{e1}$$

After the first period, at t = 1.5, each market maker updates his expectation of v_m from P_{m1} to μ_{mp}. The optimal orders in the second period are therewith given by:

$$x_{a2}^A = \beta_{a2}^A(v_a - \mu_{ap}) \qquad x_{b2}^A = \gamma_{b2}^A(\mathbb{E}(v_b|\mathcal{I}_2^A) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}})\mu_{bp})$$

$$x_{e2}^A = \beta_{e2}(\omega_a v_a - \mu_{ep}) + \omega_b \gamma_{e2}\mathbb{E}(v_b|\mathcal{I}_2^A)$$

$$P_{a2} = \kappa_{a2} + \lambda_{a2} q_{a2} \qquad P_{e2} = \kappa_{e2} + \lambda_{e2} q_{e2}$$

The constants for the asset market $\beta_{a1}^A, \gamma_{b1}^A, \beta_{a2}^A, \gamma_{b2}^A, \lambda_{a1}, \lambda_{a2}, \kappa_{a2}$ are the (numerical) solutions to the following equation system:

$$\beta_{a2}^A = \frac{1}{3\lambda_{a2}} \cdot \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}} \qquad \gamma_{b2}^A = \frac{1}{3\lambda_{b2}} \cdot \frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}} \tag{B.1}$$

$$\kappa_{a2} = \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}\mu_{ap} \qquad \lambda_{a2} = \frac{(\beta_{a2}^A + \gamma_{a2}^B(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}))\ (\tau_{vp}^a)^{-1}}{(\beta_{a2}^A + \gamma_{a2}^B(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}))^2\ (\tau_{vp}^a)^{-1} + \gamma_{a2}^{B^2}\ (\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}})^2\ var(\epsilon_{a|\mathcal{I}_2^B}) + \tau_z^{-1}} \tag{B.2}$$

$$\lambda_{a1} = \frac{\beta_{a1}^A \tau_z}{(\beta_{a1}^{A^2} + \gamma_{a1}^{B^2})\ \tau_z + \tau_v} \tag{B.3}$$

$$0 = v_a \Big[1 - \frac{2}{9\lambda_{a2}}\, IR_B^2\, \frac{1}{\tau_{vp}^a}\, (\beta_{a1}^A\, \tau_z - \gamma_{a1}^B\, \tau_z K_a)\, (1 - \frac{1}{\tau_{vp}^a}\, \tau_{ze} K_{ea}\, \beta_{e1}^A) + \tag{B.4}$$
$$\frac{2}{9\lambda_{b2}}\, IR_A^2\, \big(\frac{1}{\tau_{vp}^b}\big)^2\, (\gamma_{a1}^B \tau_z - \beta_{a1}^A \tau_z K_b)\, \tau_{ze} K_{eb}\, \beta_{e1}^A\Big] -$$
$$x_{a1}^A \Big[2\lambda_{a1} - \frac{2}{9\lambda_{a2}}\, IR_B^2\, \big(\frac{\tau_z}{\tau_{vp}^a}\big)^2\, (\beta_{a1}^A - \gamma_{a1}^B K_a)^2 - \frac{2}{9\lambda_{b2}}\, IR_A^2\, \big(\frac{\tau_z}{\tau_{vp}^b}\big)^2\, (\gamma_{a1}^B - \beta_{a1}^A K_b)^2\Big] +$$
$$x_{b1}^A \Big[\frac{2}{9\lambda_{a2}}\, IR_B^2\, \big(\frac{\tau_z}{\tau_{vp}^a}\big)^2\, (\beta_{a1}^A - \gamma_{a1}^B K_a)(\gamma_{b1}^A - \beta_{b1}^B K_a) + \frac{2}{9\lambda_{b2}}\, IR_A^2\, \big(\frac{\tau_z}{\tau_{vp}^b}\big)^2\, (\gamma_{a1}^B - \beta_{a1}^A K_b)(\beta_{b1}^B - \gamma_{b1}^A K_b)\Big]$$
$$0 = v_a \Big[- \frac{2}{9\lambda_{a2}}\, IR_B^2\, \frac{1}{\tau_{vp}^a}\, (\gamma_{b1}^A\, \tau_z - \beta_{b1}^B\, \tau_z K_a)\, (1 - \frac{1}{\tau_{vp}^a}\, \tau_{ze} K_{ea}\, \beta_{e1}^A) + \tag{B.5}$$
$$\frac{2}{9\lambda_{b2}}\, IR_A^2\, \big(\frac{1}{\tau_{vp}^b}\big)^2\, (\beta_{b1}^B - \gamma_{b1}^A\, K_b)\, \tau_z\, \tau_{ze} K_{eb}\, \beta_{e1}^A\Big] -$$
$$x_{b1}^A \Big[2\lambda_{b1} - \frac{2}{9\lambda_{a2}}\, IR_B^2\, \big(\frac{\tau_z}{\tau_{vp}^a}\big)^2\, (\gamma_{b1}^A - \beta_{b1}^B\, K_a)^2 - \frac{2}{9\lambda_{b2}}\, IR_A^2\, \big(\frac{\tau_z}{\tau_{vp}^b}\big)^2\, (\beta_{b1}^B - \gamma_{b1}^A\, K_b)^2\Big] +$$
$$x_{a1}^A \Big[\frac{2}{9\lambda_{a2}}\, IR_B^2\, \big(\frac{\tau_z}{\tau_{vp}^a}\big)^2\, (\gamma_{b1}^A - \beta_{b1}^B\, K_a)(\beta_{a1}^A - \gamma_{a1}^B K_a) + \frac{2}{9\lambda_{b2}}\, IR_A^2\, \big(\frac{\tau_z}{\tau_{vp}^b}\big)^2\, (\beta_{b1}^B - \gamma_{b1}^A\, K_b)(\gamma_{a1}^B - \beta_{a1}^A K_b)\Big]$$

where Equations (B.4) and (B.5) define the optimal order size in the first period, i.e. β_{a1}^A and γ_{b1}^A. Second-order conditions imply the following constraints on the coefficients:

$$\lambda_{a2} > 0 \tag{B.6}$$
$$\lambda_{a1} > \frac{1}{9\lambda_{a2}} IR_B^2 (\frac{\tau_z}{\tau_{vp}^a})^2 \big[(\beta_{a1}^A - \gamma_{a1}^B K_a)^2 + (\gamma_{a1}^B - \beta_{a1}^A K_b)^2\big] \tag{B.7}$$
$$(\lambda_{a1} - \frac{1}{9\lambda_{a2}} IR_B^2 (\frac{\tau_z}{\tau_{vp}^a})^2 \big[(\beta_{a1}^A - \gamma_{a1}^B K_a)^2 + (\gamma_{a1}^B - \beta_{a1}^A K_b)^2\big])^2 > \tag{B.8}$$
$$\big(\frac{2}{9\lambda_{a2}}\, IR_B^2\, \big(\frac{\tau_z}{\tau_{vp}^a}\big)^2\, (\gamma_{b1}^A - \beta_{b1}^B\, K_a)(\beta_{a1}^A - \gamma_{a1}^B K_a)\big)^2$$

The coefficients for optimal trading and pricing rules in the ETF market are given by the following system of equations:

$$\beta_{e2}^{A} = \omega_a \ \frac{1}{3\lambda_{e2}} \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}} \tag{B.9}$$

$$\gamma_{e2}^{A} = \omega_b \ \frac{1}{3\lambda_{e2}} \frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}} \tag{B.10}$$

$$\kappa_{e2} = \mu_{ep} \ \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}} \tag{B.11}$$

$$\lambda_{e2} = \frac{\omega_a(\beta_{e2}^A + \gamma_{e2}^B(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}))\ (\tau_{vp}^a)^{-1} + \omega_b(\beta_{e2}^B + \gamma_{e2}^A(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}))\ (\tau_{vp}^b)^{-1}}{(\beta_{e2}^A + \gamma_{e2}^B(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}))^2\ (\tau_{vp}^a)^{-1} + (\beta_{e2}^B + \gamma_{e2}^A(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}))^2\ (\tau_{vp}^b)^{-1} + \mathcal{E}_{ve} + \tau_{ze}^{-1}} \tag{B.12}$$

$$\lambda_{e1} = \frac{(\omega_a\beta_{e1}^A + \omega_b\beta_{e1}^B)\tau_{ze}}{(\beta_{e1}^{A^2} + \beta_{e1}^{B^2})\tau_{ze} + \tau_v} \tag{B.13}$$

$$\begin{aligned} 0 =& v_a\Big[\omega_a - \frac{1}{9\lambda_{e2}}\ IR_B^2(\omega_a\frac{K_{ea}}{\tau_{vp}^a} + \omega_b\frac{K_{eb}}{\tau_{vp}^b})\Big(\omega_a - \omega_a\frac{1}{\tau_{vp}^a}[(\beta_{a1}^{A^2} + \gamma_{b1}^{A^2})\tau_z - (\gamma_{a1}^B\beta_{a1}^A + \gamma_{b1}^A\beta_{b1}^B)K_a] - \\ & \omega_b\frac{1}{\tau_{vp}^b}[(\gamma_{a1}^B\beta_{a1}^A + \gamma_{b1}^A\beta_{b1}^B)\tau_z - (\beta_{a1}^{A^2} + \gamma_{b1}^{A^2})K_b]\Big)\Big] - \\ & x_{e1}^A\Big[2\lambda_{e1} - \frac{2}{9\lambda_{e2}}IR_B^2\ (\omega_a\frac{K_{ea}}{\tau_{vp}^a} + \omega_b\frac{K_{eb}}{\tau_{vp}^b})^2\Big] \end{aligned} \tag{B.14}$$

where Equation (B.14) defines the optimal trading in t = 1. The second order conditions imply the following constraints:

$$\lambda_{e2} > 0 \tag{B.15}$$

$$\lambda_{e1} > \frac{1}{9\lambda_{e2}}IR_B^2\ (\omega_a\frac{K_{ea}}{\tau_{vp}^a} + \omega_b\frac{K_{eb}}{\tau_{vp}^b})^2 \tag{B.16}$$

The equations for market b and the trader informed about v_b in both, the asset market and the ETF, follow by symmetry. Note[12] that $\tau_{v|\mathcal{I}_2^A}$ denotes the conditional precision of the

[12] The equations for these quantities can be found in the Proof.

signal of speculator A after observing the t = 1 order flows in all markets, τ_{vp}^{a} denotes the conditional precision of the market maker upon observing the t = 1 order flows in all markets and $IR_B = \frac{1+\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}{1+\frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}$. Define:

$$K_a = \frac{(\beta_{a1}^{A}\ \gamma_{a1}^{B} + \beta_{b1}^{B}\ \gamma_{b1}^{A})\tau_z + \beta_{e1}^{A}\beta_{e1}^{B}\tau_{ze}}{\beta_{e1}^{B^2}\tau_{ze} + (\tau_v + (\gamma_{a1}^{B^2} + \beta_{b1}^{B^2})\tau_z)}$$

$$K_{ea} = \frac{\beta_{e1}^{A}(\tau_v + (\beta_{b1}^{B^2} + \gamma_{a1}^{B^2})\tau_z) - \beta_{e1}^{B}(\beta_{a1}^{A}\ \gamma_{a1}^{B} + \beta_{b1}^{B}\ \gamma_{b1}^{A})\tau_z}{\beta_{e1}^{B^2}\tau_{ze} + (\tau_v + (\gamma_{a1}^{B^2} + \beta_{b1}^{B^2})\tau_z)}$$

C Proofs

Proof of Proposition 2

The proof proceeds in two steps. First, I solve the filtration problem of the speculators and market makers by assuming an arbitrary strategy profile $\beta_{at}^{A}, \beta_{bt}^{B}, \gamma_{bt}^{A}, \gamma_{at}^{B}, \beta_{e,t}^{A}, \beta_{e,t}^{B}, \lambda_{at}, \lambda_{bt}, \lambda_{e,t}$ $\forall t \in \{1, 2\}$. Next, I solve the optimization problem of the speculators for the asset markets and the ETF.

Step 1: First, let us consider the updating problem of a speculator knowing v_a. Here, it is irrelevant whether the speculator trades in the ETF or in the underlying asset markets, as the information structure is the same. Hence, let us consider asset speculator A going forward, unless stated otherwise.

Her information sets are given by $\mathcal{I}_1^A = \{v_a\}$, $\mathcal{I}_2^A = \{v_a, q_{a,1}, q_{b,1}, q_{e,1}\}$. It is $\mathbb{E}(v_b|\mathcal{I}_1^A) = 0$. Knowing v_a and her own demand in the markets a and b, the three order flows provide her with three signals about v_b, given by:

$$s_{b,1} = \frac{q_{a,1} - x_{a,1}^{A}}{\gamma_{a,1}^{B}} v_b + \frac{z_{a,1}}{\gamma_{a,1}^{B}}$$

$$s_{b,2} = \frac{q_{b,1} - x_{b,1}^{A}}{\beta_{b,1}^{B}} v_b + \frac{z_{b,1}}{\beta_{b,1}^{B}}$$

$$s_{b,3} = \frac{q_{e,1} - \beta_{e,1}^{A} v_a}{\beta_{e,1}^{B}} v_b + \frac{z_{e,1}}{\beta_{e,1}^{B}}$$

With this information, her posterior expectation and variance of v_b are:

$$Var(v_b|\mathcal{I}_2^A) = \frac{1}{\tau_{v|\mathcal{I}_2^A}} = \left(\tau_v + (\gamma_{a,1}^{B^2} + \beta_{b,1}^{B^2})\tau_z + \beta_{e,1}^{B^2}\tau_{z,e}\right)^{-1} \tag{C.1}$$

$$\mathbb{E}(v_b|\mathcal{I}_2^A) = \frac{1}{\tau_{v|\mathcal{I}_2^A}}\left((\beta_{b,1}^B q_{b,1} + \gamma_{a,1}^B q_{a,1})\tau_z - (\beta_{b,1}^B\ x_{b,1}^A + \gamma_{a,1}^B x_{a,1}^A)\tau_z + \beta_{e,1}^B \tau_{z,e}(q_{e,1} - x_{e,1}^A)\right) \tag{C.2}$$

$$= \left(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}\right)v_b + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}\epsilon_{\mathcal{I}_2^A}$$

where $\epsilon_{\mathcal{I}_2^A} \sim \mathcal{N}(0, \frac{1}{\tau_v^2}((\gamma_{a,1}^{B^2} + \beta_{b,1}^{B^2})\tau_z + \beta_{e,1}^{B^2}\tau_{z,e}))$ is the signal error. By symmetry, the same updating holds for the speculator informed about v_b.

Next, consider the learning problem of the market maker who is setting prices for v_a. His information sets are given by $\mathcal{I}_1^{MM} = \{q_{a,1}\}$, $\mathcal{I}_{1.5}^{MM} = \{q_{a,1}, q_{b,1}, q_{e,1}\}$ and $\mathcal{I}_2^{MM} = \{q_{a,1}, q_{b,1}, q_{e,1}, q_{a,2}\}$. In t=1, his signal is

$$s_{a,1} = \frac{q_{a,1}}{\beta_{a,1}^A} v_a + \frac{\gamma_{a,1}^B\ v_b + z_{a,1}}{\beta_{a,1}^A}$$

which leads to the conditional expectation and hence $P_{a,1}$ as

$$P_{a,1} = \mathbb{E}(v_a|s_{a,1}) = \frac{\beta_{a,1}^A \tau_z}{(\beta_{a,1}^{A^2} + \gamma_{a,1}^{B^2})\ \tau_z + \tau_v}\ q_{a,1} = \lambda_{a,1}\ q_{a,1} \tag{C.3}$$

and $\kappa_{a,1} = 0$.

After the first period, the market maker can observe the prices in all other markets. The price functions in all markets can be inverted to yield the aggregate order flow in all other markets. This provides the market maker with three signals about v_a

$$s_{a,1} = \frac{q_{a,1}}{\beta^A_{a,1}} v_a + \frac{\gamma^B_{a,1}\ v_b + z_{a,1}}{\beta^A_{a,1}}$$
$$s_{a,2} = \frac{q_{b,1}}{\gamma^A_{b,1}} v_a + \frac{\beta^B_{b,1}\ v_b + z_{b,1}}{\gamma^A_{b,1}}$$
$$s_{a,3} = \frac{q_{e,1}}{\beta^A_{e,1}} v_a + \frac{\beta^B_{e,1}\ v_b + z_{e,1}}{\beta^A_{e,1}}$$

The noise in all these signals is correlated, as they depend on v_b. Therefore, I use the Projection Theorem to derive the posterior variance and expectation of the market maker. They are given by:

$$Var(v_a|\mathcal{I}^{MM}_{1.5}) = \frac{1}{\tau^a_{vp}} = \Big(\tau_v + (\gamma^{A^2}_{b,1} + \beta^{A^2}_{a,1})\tau_z + \beta^{A^2}_{e,1}\tau_{z,e}\frac{(\tau_v + (\gamma^{B^2}_{a,1} + \beta^{B^2}_{b,1})\tau_z)}{\beta^{B^2}_{e,1}\tau_{z,e} + (\tau_v + (\gamma^{B^2}_{a,1} + \beta^{B^2}_{b,1})\tau_z)} \tag{C.4}$$
$$- (\beta^A_{a,1}\ \gamma^B_{a,1} + \beta^B_{b,1}\ \gamma^A_{b,1})\tau_z \frac{(\beta^A_{a,1}\ \gamma^B_{a,1} + \beta^B_{b,1}\ \gamma^A_{b,1})\tau_z + 2\beta^A_{e,1}\ \beta^B_{e,1}\tau_{z,e}}{\beta^{B^2}_{e,1}\tau_{z,e} + (\tau_v + (\gamma^{B^2}_{a,1} + \beta^{B^2}_{b,1})\tau_z)}\Big)^{-1}$$
$$\mathbb{E}(v_a|\mathcal{I}^{MM}_{1.5}) = \mu_{ap} = \frac{1}{\tau^a_{vp}} * \big((\beta^A_{a,1}q_{a,1} + \gamma^A_{b,1}q_{b,1})\tau_z - (\beta^B_{b,1}q_{b,1} + \gamma^B_{a,1}q_{a,1})\ \tau_z\ K_a + q_{e,1}\tau_{z,e}K_{ea}\big) \tag{C.5}$$

with

$$K_a = \frac{(\beta^A_{a,1}\ \gamma^B_{a,1} + \beta^B_{b,1}\ \gamma^A_{b,1})\tau_z + \beta^A_{e,1}\beta^B_{e,1}\tau_{z,e}}{\beta^{B^2}_{e,1}\tau_{z,e} + (\tau_v + (\gamma^{B^2}_{a,1} + \beta^{B^2}_{b,1})\tau_z)}$$
$$K_{ea} = \frac{\beta^A_{e,1}(\tau_v + (\beta^{B^2}_{b,1} + \gamma^{B^2}_{a,1})\tau_z) - \beta^B_{e,1}(\beta^A_{a,1}\ \gamma^B_{a,1} + \beta^B_{b,1}\ \gamma^A_{b,1})\tau_z}{\beta^{B^2}_{e,1}\tau_{z,e} + (\tau_v + (\gamma^{B^2}_{a,1} + \beta^{B^2}_{b,1})\tau_z)}$$

Finally, in t = 2, the market maker observes the order flow in his market $q_{a,2}$ again. Let us use the flexible equilibrium conjecture that $x^A_{a,2} = \alpha^A_{a,2} + \beta^A_{a,2}v_a$ and $x^B_{a,2} = \alpha^B_{a,2} + \gamma^B_{a,2}\mathbb{E}(v_a|\mathcal{I}^B_2)$

and

$$
\begin{aligned}
q_{a,2} &= x^A_{a,2} + x^B_{a,2} + z_{a,2} \\
&= \alpha^A_{a,2} + \beta^A_{a,2} v_a + \alpha^B_{a,2} + \gamma^B_{a,2} \mathbb{E}(v_a | \mathcal{I}^B_2) + z_{a,2} \\
&= \alpha^A_{a,2} + \beta^A_{a,2} v_a + \alpha^B_{a,2} + \gamma^B_{a,2} \Big((1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}) v_a + \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}} \epsilon_{\mathcal{I}^B_2} \Big) \\
&= \alpha^A_{a,2} + \alpha^B_{a,2} + \Big(\beta^A_{a,2} + \gamma^B_{a,2} (1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}) \Big) v_a + \gamma^B_{a,2} \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}} \epsilon_{\mathcal{I}^B_2} + z_{a,2}
\end{aligned}
$$

Hence, the signal is given by:

$$
s_{a,4} = \frac{q_{a,2} - \alpha^A_{a,2} - \alpha^B_{a,2}}{\beta^A_{a,2} + \gamma^B_{a,2} \left(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}\right)} v_a + \frac{\gamma^B_{a,2} \left(\frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}} \right) \epsilon_{\mathcal{I}^B_2} + z_{a,2}}{\beta^A_{a,2} + \gamma^B_{a,2} \left(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}\right)}
$$

with this, I find

$$
P_{a,2} = \mathbb{E}(v_a | s_{a,1}, s_{a,2}, s_{a,3}, s_{a,4}) = \kappa_{a,2} + \lambda_{a,2} q_{a,2} \tag{C.6}
$$

$$
\lambda_{a,2} = \frac{(\beta^A_{a,2} + \gamma^B_{a,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}))\ \frac{1}{\tau^a_{vp}}}{(\beta^A_{a,2} + \gamma^B_{a,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}))^2\ \frac{1}{\tau^a_{vp}} + \gamma^{B^2}_{a,2}\ (\frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}})^2\ var(\epsilon_{a|\mathcal{I}^B_2}) + \tau_z^{-1}} \tag{C.7}
$$

$$
\kappa_{a,2} = \mu_{ap} - \lambda_{a,2} \big[\alpha^A_{a,2} + \alpha^B_{a,2} + (\beta^A_{a,2} + \gamma^B_{a,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}))\ \mu_{ap} \big] \tag{C.8}
$$

Using symmetry, it can be shown that the updating for the market maker setting prices for v_b yields equivalent equations. *The pricing functions for the ETF* are found using the same reasoning and yield:

$$\lambda_{e,1} = \frac{(\omega_a \beta^A_{e,1} + \omega_b \beta^B_{e,1})\tau_{z,e}}{(\beta^{A2}_{e,1} + \beta^{B2}_{e,1})\tau_{z,e} + \tau_v}$$

$$\kappa_{e,1} = 0$$

$$\mu_{ep} = \omega_a \mu_{ap} + \omega_b \mu_{bp}$$

$$\kappa_{e,2} = \mu_{ep} - \lambda_{e,2}(\alpha^A_{e,2} + \alpha^B_{e,2} + (\beta^A_{e,2} + \gamma^B_{e,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}))\mu_{ap} + (\beta^B_{e,2} + \gamma^A_{e,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}}))\mu_{bp})$$

$$\lambda_{e,2} = \frac{\omega_a(\beta^A_{e,2} + \gamma^B_{e,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}))\ \frac{1}{\tau^a_{vp}} + \omega_b(\beta^B_{e,2} + \gamma^A_{e,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}}))\ \frac{1}{\tau^b_{vp}}}{(\beta^A_{e,2} + \gamma^B_{e,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}))^2\ \frac{1}{\tau^a_{vp}} + (\beta^B_{e,2} + \gamma^A_{e,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}}))^2\ \frac{1}{\tau^b_{vp}} + \mathcal{E}_{ve} + \tau^{-1}_{z,e}}$$

with

$$\mathcal{E}_{ve} = (\gamma^B_{e,2} \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}})^2 var(\epsilon_a | \mathcal{I}^B_2) + (\gamma^A_{e,2} \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})^2 var(\epsilon_a | \mathcal{I}^A_2)$$

This describes the pricing rules in all asset markets and the ETF.

Step 2: Next, I solve the optimization problem of the *asset speculator* A using backwards induction. The solution to the optimization problem of the asset speculator B follows by symmetry. In t = 2, the optimization problem of speculator A is given by:

$$\mathbb{E}(x^A_{a,2}\ (v_a - P_{a,2})\ +\ x^A_{b,2}\ (v_b - P_{b,2})\ |\mathcal{I}^A_2)$$

This can be optimized separately for $x^A_{a,2}$ and $x^A_{b,2}$, as there is no feedback-effect between these two markets after the second period.[13] Inserting the conjectured pricing functions $P_{m2} = \kappa_{m2} + \lambda_{m2} q_{m2}\ \forall m \in \{a, b\}$ yields:

$$x^A_{a,2} = (\frac{1}{2\lambda_{a,2}} - \frac{1}{2}\gamma^B_{a,2}(1 - \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}))\ v_a - (\frac{1}{2}\alpha^B_{a,2} + \frac{1}{2\lambda_{a,2}}\kappa_{a,2})$$

$$x^A_{b,2} = (\frac{1}{2\lambda_{b,2}} - \frac{1}{2}\beta^B_{b,2}))\ \mathbb{E}(v_b|\mathcal{I}^A_2) - (\frac{1}{2}\alpha^B_{b,2} + \frac{1}{2\lambda_{b,2}}\kappa_{b,2})$$

[13] Formally, the cross-derivative of the expected profit $\frac{\partial \mathbb{E}(\pi^A_2 | \mathcal{I}^A_2)}{\partial x^A_{a,2} \partial x^A_{b,2}}$ is zero.

The second-order conditions imply $\lambda_{a,2} > 0$ and $\lambda_{b,2} > 0$. Imposing symmetry then yields:

$$\beta^A_{a,2} = \frac{1}{3\lambda_{a,2}} \cdot \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}} \tag{C.9}$$

$$\gamma^B_{a,2} = \frac{1}{3\lambda_{a,2}} \cdot \frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}} \tag{C.10}$$

$$\alpha^A_{a,2} = \alpha^B_{a,2} = -\frac{1}{3\lambda_{a,2}}\kappa_{a,2} \tag{C.11}$$

$$\kappa_{a,2} = \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}}\mu_{ap} \tag{C.12}$$

Next, we can rewrite the optimal orders $x^A_{a,2} = \alpha^A_{a,2} + \beta^A_{a,2}v_a$ and $x^A_{b,2} = \alpha^A_{b,2} + \beta^A_{b,2}\mathbb{E}(v_a|\mathcal{I}^A_2)$ to yield:

$$x^A_{a,2} = \beta^A_{a,2}\,(v_a - \mu_{ap}) \tag{C.13}$$

$$x^A_{b,2} = \gamma^A_{b,2}\,(\mathbb{E}(v_b|\mathcal{I}^A_2) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}}) \cdot \mu_{bp}) \tag{C.14}$$

Note that this is the optimal order size as given in Proposition 2. Finally, we can insert these solutions to calculate the expected profit in the respective market and obtain the value function of asset speculator A. This is:

$$\mathbb{E}(x^A_{a,2}\,(v_a - P_{a,2})) = \mathbb{E}\left[\frac{1}{9\lambda_{a,2}} \cdot \left(\frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}}\right)^2 (v_a - \mu_{ap})^2\right] = \mathbb{E}(V^A_a(x^A_{a,1}, x^A_{b,1}))$$

$$\mathbb{E}(x^A_{b,2}\,(v_b - P_{b,2})) = \mathbb{E}\left[\frac{1}{9\lambda_{b,2}} \cdot \left(\frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}}}\right)^2 (\mathbb{E}(v_b|\mathcal{I}^A_2) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\mu_{bp})^2\right] = \mathbb{E}(V^A_b(x^A_{a,1}, x^A_{b,1}))$$

With the value functions, we can now solve for the *optimization problem of asset specu-*

lator A in t = 1. For this, I first define the information ratios to save on notation as

$$IR_A = \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}}$$
$$IR_B = \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}$$

The optimization problem of the asset speculator A in t=1 is then given by:

$$\underset{x_{a,1}^A, x_{b,1}^A}{\operatorname{argmax}} \mathbb{E}(x_{a,1}^A(v_a - P_{a,1}) + x_{b,1}^A(v_b - P_{b,1}) + V_a^A(x_{a,1}^A, x_{b,1}^A) + V_b^A(x_{a,1}^A, x_{b,1}^A))|v_a)$$

Note that a simultaneous optimization is necessary as $V_a^A(x_{a,1}^A, x_{b,1}^A)$ and $V_b^A(x_{a,1}^A, x_{b,1}^A)$ are functions of both, $x_{a,1}^A$ and $x_{b,1}^A$ The first order condition with respect to $x_{a,1}^A$ yields

$$0 = v_a - 2\lambda_{a,1}\, x_{a,1}^A + \mathbb{E}\Big(\frac{\partial V_a^A(x_{a,1}^A, x_{b,1}^A)}{\partial x_{a,1}^A}|v_a\Big) + \mathbb{E}\Big(\frac{\partial V_b^A(x_{a,1}^A, x_{b,1}^A)}{\partial x_{a,1}^A}|v_a\Big) \tag{C.15}$$

with

$$\mathbb{E}\Big(\frac{\partial V_a^A(x_{a,1}^A, x_{b,1}^A)}{\partial x_{a,1}^A}|v_a\Big) = \frac{2}{9\lambda_{a,2}}\, IR_B^2\, (v_a \;-\; \mathbb{E}(\mu_{ap}|v_a))\, \big(-\frac{1}{\tau_{vp}^a}\,(\beta_{a,1}^A\tau_z - \gamma_{a,1}^B\tau_z K_a)\big)$$
$$\mathbb{E}\Big(\frac{\partial V_b^A(x_{a,1}^A, x_{b,1}^A)}{\partial x_{a,1}^A}|v_a\Big) = \frac{2}{9\lambda_{b,2}}\, IR_A^2\, \mathbb{E}(\mu_{bp}|v_a)\, \big(\frac{1}{\tau_{vp}^b}\,(\gamma_{a,1}^B\tau_z - \beta_{a,1}^A\tau_z K_b)\big)$$
$$\mathbb{E}(\mu_{ap}|v_a) = \frac{1}{\tau_{vp}^a}\big((\beta_{a,1}^A x_{a,1}^A + \gamma_{b,1}^A x_{b,1}^A)\tau_z - (\gamma_{a,1}^B x_{a,1}^A + \beta_{b,1}^B x_{b,1}^A)\tau_z K_a + \beta_{e,1}^A v_a \tau_{z,e} K_{ea}\big)$$
$$\mathbb{E}(\mu_{bp}|v_a) = \frac{1}{\tau_{vp}^b}\big((\beta_{b,1}^B x_{b,1}^A + \gamma_{a,1}^B x_{a,1}^A)\tau_z - (\gamma_{b,1}^A x_{b,1}^A + \beta_{a,1}^A x_{a,1}^A)\tau_z K_b + \beta_{e,1}^A v_a \tau_{z,e} K_{eb}\big)$$

Putting all the terms together eventually yields:

$$0 = v_a \Bigg[1 - \frac{2}{9\lambda_{a,2}}\, IR_B^2\, \frac{1}{\tau_{vp}^a}\, (\beta_{a,1}^A\, \tau_z - \gamma_{a,1}^B\, \tau_z K_a)\, (1 - \frac{1}{\tau_{vp}^a}\, \tau_{z,e} K_{ea}\, \beta_{e,1}^A) +$$
$$\frac{2}{9\lambda_{b,2}}\, IR_A^2\, (\frac{1}{\tau_{vp}^b})^2\, (\gamma_{a,1}^B \tau_z - \beta_{a,1}^A \tau_z K_b)\, \tau_{z,e} K_{eb}\, \beta_{e,1}^A\Bigg] -$$
$$x_{a,1}^A \Bigg[2\lambda_{a,1} - \frac{2}{9\lambda_{a,2}}\, IR_B^2\, (\frac{\tau_z}{\tau_{vp}^a})^2\, (\beta_{a,1}^A - \gamma_{a,1}^B K_a)^2 - \frac{2}{9\lambda_{b,2}}\, IR_A^2\, (\frac{\tau_z}{\tau_{vp}^b})^2\, (\gamma_{a,1}^B - \beta_{a,1}^A K_b)^2\Bigg] +$$
$$x_{b,1}^A \Bigg[\frac{2}{9\lambda_{a,2}}\, IR_B^2\, (\frac{\tau_z}{\tau_{vp}^a})^2\, (\beta_{a,1}^A - \gamma_{a,1}^B K_a)(\gamma_{b,1}^A - \beta_{b,1}^B K_a) + \frac{2}{9\lambda_{b,2}}\, IR_A^2\, (\frac{\tau_z}{\tau_{vp}^b})^2\, (\gamma_{a,1}^B - \beta_{a,1}^A K_b)(\beta_{b,1}^B - \gamma_{b,1}^A K_b)\Bigg]$$

Hence, the optimal order size in the own market ($x_{a,1}^A$) depends on the liquidation value of the asset (v_a) as well as on the trading behaviour in the other market ($x_{b,1}^A$). Using the same arguments for the derivative of the expected profit with respect to $x_{b,1}^A$ yields

$$0 = v_a \Bigg[- \frac{2}{9\lambda_{a,2}}\, IR_B^2\, \frac{1}{\tau_{vp}^a}\, (\gamma_{b,1}^A\, \tau_z - \beta_{b,1}^B\, \tau_z K_a)\, (1 - \frac{1}{\tau_{vp}^a}\, \tau_{z,e} K_{ea}\, \beta_{e,1}^A) +$$
$$\frac{2}{9\lambda_{b,2}}\, IR_A^2\, (\frac{1}{\tau_{vp}^b})^2\, (\beta_{b,1}^B - \gamma_{b,1}^A\, K_b)\, \tau_z\, \tau_{z,e} K_{eb}\, \beta_{e,1}^A\Bigg] -$$
$$x_{b,1}^A \Bigg[2\lambda_{b,1} - \frac{2}{9\lambda_{a,2}}\, IR_B^2\, (\frac{\tau_z}{\tau_{vp}^a})^2\, (\gamma_{b,1}^A - \beta_{b,1}^B\, K_a)^2 - \frac{2}{9\lambda_{b,2}}\, IR_A^2\, (\frac{\tau_z}{\tau_{vp}^b})^2\, (\beta_{b,1}^B - \gamma_{b,1}^A\, K_b)^2\Bigg] +$$
$$x_{a,1}^A \Bigg[\frac{2}{9\lambda_{a,2}}\, IR_B^2\, (\frac{\tau_z}{\tau_{vp}^a})^2\, (\gamma_{b,1}^A - \beta_{b,1}^B\, K_a)(\beta_{a,1}^A - \gamma_{a,1}^B K_a) + \frac{2}{9\lambda_{b,2}}\, IR_A^2\, (\frac{\tau_z}{\tau_{vp}^b})^2\, (\beta_{b,1}^B - \gamma_{b,1}^A\, K_b)(\gamma_{a,1}^B - \beta_{a,1}^A K_b)\Bigg]$$

Simultaneously solving these equations for $x_{a,1}^A$ and $x_{b,1}^A$, I find $x_{a,1}^A$ and $x_{b,1}^A$ as a function of v_a, respectively. Especially, it is $x_{b,1}^A \neq 0$. Moreover, the equilibrium is symmetric such that we can find the expressions for the optimal orders of asset trader B in the same way. In addition, note that I focus on ETFs with equally weighted assets, such that $\omega_a = \omega_b = 0.5$. Therefore, I find that $IR_A = IR_B$, $\tau_{vp}^a = \tau_{vp}^b$ and hence $\lambda_{a,2} = \lambda_{b,2}$.

Second order conditions require that the Hessian

$$\mathcal{H}_1 = 2 \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$$

with

$$h_{11} = \left[\lambda_{a,1} \;-\; \frac{1}{9\lambda_{a,2}}\; IR_B^2 \;\big(\frac{\tau_z}{\tau_{vp}^a}\big)^2 \;(\beta_{a,1}^A - \;\gamma_{a,1}^B K_a)^2 - \frac{1}{9\lambda_{b,2}}\; IR_A^2 \;\big(\frac{\tau_z}{\tau_{vp}^b}\big)^2 \;(\gamma_{a,1}^B - \beta_{a,1}^A K_b)^2\right]$$

$$h_{21} = h_{12} = \left[\frac{1}{9\lambda_{a,2}}\; IR_B^2 \;\big(\frac{\tau_z}{\tau_{vp}^a}\big)^2 \;(\gamma_{b,1}^A - \;\beta_{b,1}^B\; K_a)(\beta_{a,1}^A - \gamma_{a,1}^B K_a) + \frac{1}{9\lambda_{b,2}}\; IR_A^2 \;\big(\frac{\tau_z}{\tau_{vp}^b}\big)^2 \;(\beta_{b,1}^B - \gamma_{b,1}^A\; K_b)(\gamma_{a,1}^B - \beta_{a}$$

$$h_{22} = \left[\lambda_{b,1} - \frac{1}{9\lambda_{a,2}}\; IR_B^2 \;\big(\frac{\tau_z}{\tau_{vp}^a}\big)^2 \;(\gamma_{b,1}^A - \;\beta_{b,1}^B\; K_a)^2 - \frac{1}{9\lambda_{b,2}}\; IR_A^2 \;\big(\frac{\tau_z}{\tau_{vp}^b}\big)^2 \;(\beta_{b,1}^B - \gamma_{b,1}^A\; K_b)^2\right]$$

is negative definite. This, in turn, is equivalent to requiring that

$$h_{11} < 0$$

$$h_{22} < 0$$

$$(h_{11}h_{22} - h_{12}^2) > 0$$

which implies that

$$\lambda_{a,1} > \frac{1}{9\lambda_{a,2}} IR_B^2 (\frac{\tau_z}{\tau_{vp}^a})^2 \big[(\beta_{a,1}^A - \gamma_{a,1}^B K_a)^2 + (\gamma_{a,1}^B - \beta_{a,1}^A K_b)^2\big] \tag{C.16}$$

$$(\lambda_{a,1} - \frac{1}{9\lambda_{a,2}} IR_B^2 (\frac{\tau_z}{\tau_{vp}^a})^2 \big[(\beta_{a,1}^A - \gamma_{a,1}^B K_a)^2 + (\gamma_{a,1}^B - \beta_{a,1}^A K_b)^2\big])^2 > \tag{C.17}$$

$$\big(\frac{2}{9\lambda_{a,2}}\; IR_B^2 \;\big(\frac{\tau_z}{\tau_{vp}^a}\big)^2 \;(\gamma_{b,1}^A - \;\beta_{b,1}^B\; K_a)(\beta_{a,1}^A - \gamma_{a,1}^B K_a)\big)^2$$

Repeating the analysis for the ETF speculator informed about v_a yields the following results:

$$\beta_{e,2}^A = \omega_a \ \frac{1}{3\lambda_{e,2}} \frac{1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}$$

$$\gamma_{e,2}^A = \omega_b \ \frac{1}{3\lambda_{e,2}} \frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}}$$

$$\alpha_{e,2}^A = \alpha_{e,2}^B = -\frac{1}{3\kappa_{e,2}}$$

$$\kappa_{e,2} = \mu_{ep} \ IR_A$$

and the second-order condition $\lambda_{e,2} > 0$

Using equal weights of the assets in the ETF, we can rewrite

$$\beta_{e,2}^A = \omega_a \beta_{e,2} = \beta_{e,2}^B$$

$$\beta_{e,2}^A = \omega_b \gamma_{e,2} = \gamma_{e,2}^B$$

which yields the results in Proposition 2 as:

$$x_{e,2}^A = \beta_{e,2}(\omega_a v_a - \mu_{ep}) + \omega_b \gamma_{e,2} \mathbb{E}(v_b|\mathcal{I}_2^A)$$

$$x_{e,2}^B = \beta_{e,2}(\omega_b v_b - \mu_{ep}) + \omega_a \gamma_{e,2} \mathbb{E}(v_a|\mathcal{I}_2^B)$$

The value function of ETF trader A is given by:

$$V_e^A(x_{e,1}^A) = \frac{1}{9\lambda_{e,2}} \ IR_B^2 \ (\omega_a v_a - \mu_{ep})^2 + \frac{1}{9\lambda_{e,2}} \ \big(\frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}}\big)^2 \ \omega_b^2 \ \mathbb{E}(v_b|\mathcal{I}_2^A)^2$$

which yields the following first order condition for $x^A_{e,1}$:

$$0 = v_a\Big[\omega_a - \frac{1}{9\lambda_{e,2}}\ IR_B^2(\omega_a\frac{K_{ea}}{\tau^a_{vp}} + \omega_b\frac{K_{eb}}{\tau^b_{vp}})\Big(\omega_a - \omega_a\frac{1}{\tau^a_{vp}}[(\beta^{A^2}_{a,1} + \gamma^{A^2}_{b,1})\tau_z - (\gamma^B_{a,1}\beta^A_{a,1} + \gamma^A_{b,1}\beta^B_{b,1})K_a] -$$

$$\omega_b\frac{1}{\tau^b_{vp}}[(\gamma^B_{a,1}\beta^A_{a,1} + \gamma^A_{b,1}\beta^B_{b,1})\tau_z - (\beta^{A^2}_{a,1} + \gamma^{A^2}_{b,1})K_b]\Big)\Big] -$$

$$x^A_{e,1}\Big[2\lambda_{e,1} - \frac{2}{9\lambda_{e,2}}IR_B^2\ (\omega_a\frac{K_{ea}}{\tau^a_{vp}} + \omega_b\frac{K_{eb}}{\tau^b_{vp}})^2\Big]$$

From the FOC, we can see that $\beta^A_{e,1} = \beta^B_{e,1}$ iff $\omega_a = \omega_b$, i.e. when the ETF represents the assets with equal weights. The second order condition implies that

$$\lambda_{e,1} > \frac{1}{9\lambda_{e,2}}IR_B^2\ (\omega_a\frac{K_{ea}}{\tau^a_{vp}} + \omega_b\frac{K_{eb}}{\tau^b_{vp}})^2$$

Overall, this yields the results in the proposition and completes the proof.

Proof of Proposition 3

First, consider the market without an ETF. It is $P_{m2} = \kappa_{m2} + \lambda_{m2}\ q_{m2} = P_{m1} + \frac{1}{2}\sqrt{\frac{\tau_z}{\tau_{v_{m1}}}}\ q_{m2}$ and hence

$$\begin{aligned} cov(P_{a,2}, P_{b,2}) &= cov(P_{a1}, P_{b1}) + \lambda_{a,2}cov(q_{a,2}, P_{b1}) + \lambda_{b,2}cov(q_{b,2}, P_{a1}) + \lambda_{a,2}\lambda_{b,2}cov(q_{a,2}, q_{b,2}) \\ &= cov(P_{a1}, P_{b1}) \\ &= \lambda_{a1}\lambda_{b1}cov(q_{a1}, q_{b1}) \\ &= 0 \end{aligned}$$

Next, consider the market with an ETF. Here, it is $P^{ETF}_{m2} = \kappa^{ETF}_{m2} + \lambda^{ETF}_{m2}\ q^{ETF}_{m2} = IR_T\ \mu_{mp} + \lambda_{m2}\ q_{m2}$ with $T \in \{A, B\}$, respectively. Let T^c denote the complementary element to T in

$\{A, B\}$. Note that

$$
\begin{aligned}
q_{m2} &= x^A_{m2} + x^B_{m2} + z_{m2} \\
&= \beta^T_{m2}(v_m - \mu_{mp}) + \gamma^{T^c}_{m2}(\mathbb{E}(v_m|\mathcal{I}^{T'}_2) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^{T'}_2}})\mu_{mp}) + z_{m2}
\end{aligned}
$$

and

$$
\begin{aligned}
\lambda^{ETF}_{m2}\ q^{ETF}_{m2} &= \frac{1}{3} IR_{T^c}(v_m - \mu_{mp}) + \frac{1}{3}\frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^{T^c}_2}}}(\mathbb{E}(v_m|\mathcal{I}^{T^c}_2) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^{T^c}_2}})\mu_{mp}) + \lambda_{m2} z_{m2} \\
&= \frac{2}{3}\frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^{T^c}_2}}} v_m - \frac{2}{3} IR_{T^c}\mu_{mp} + \frac{\frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^{T^c}_2}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^{T^c}_2}}}\epsilon_{\mathcal{I}^{T^c}_2} + \lambda_{m2} z_{m2} \\
\kappa^{ETF}_{m2} &= IR_{T^c}\ \mu_{mp}
\end{aligned}
$$

Hence, we have:

$$
\begin{aligned}
cov(P^{ETF}_{a,2}, P^{ETF}_{b,2}) &= IR_A\ IR_B\ cov(\mu_{ap}, \mu_{bp}) + IR_A\ cov(\mu_{bp}, \lambda_{a,2}q_{a,2}) + IR_B\ cov(\mu_{ap}, \lambda_{b,2}q_{b,2}) + cov(\lambda_{a,2}q_{a,2}, \lambda \\
&= IR^2_B\ cov(\mu_{ap}, \mu_{bp}) + 2\ IR_B\ cov(\mu_{bp}, \lambda_{a,2}q_{a,2}) + cov(\lambda_{a,2}q_{a,2}, \lambda_{b,2}q_{b,2}) \\
&= IR^2_B\ cov(\mu_{ap}, \mu_{bp}) + 2\ IR_B\ \Big[\frac{2}{3}\frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^{T'}_2}}} cov(\mu_{bp}, v_a) - \frac{2}{3} IR_B cov(\mu_{ap}, \mu_{bp}) + \\
&\quad \frac{\frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^{T^c}_2}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}^{T^c}_2}}} cov(\epsilon_{\mathcal{I}^B_2}, \mu_{bp})\Big] + cov(\lambda_{a,2}q_{a,2}, \lambda_{b,2}q_{b,2})
\end{aligned}
$$

where I used the symmetry of the equilibrium from line 1 to line 2. Again, using symmetry and writing $\beta_{a1}^A = \beta_{b1}^B = \beta_1$ and $\gamma_{a1}^B = \gamma_{b1}^A = \gamma_1$, I find:

$$
\begin{aligned}
cov(\mu_{ap}, \mu_{bp}) &= (\frac{1}{\tau_{vp}^a})^2 \Big[((\beta_1 - \gamma_1 K_a)^2 + (\gamma_1 - \beta_1 K_a)^2)\tau_z^2 \; cov(q_{a1}, q_{b1}) + \\
&\quad ((\beta_1 - \gamma_1 K_a) + (\gamma_1 - \beta_1 K_a))\tau_{ze}\tau_z K_{ea} \; (cov(q_{e1}, q_{a1}) + cov(q_{e1}, q_{b1})) \\
&\quad ((\beta_1 - \gamma_1 K_a)(\gamma_1 - \beta_1 K_a)\tau_z^2 \; (var(q_{a1}) + var(q_{b1}))) + \tau_{ze}^2 K_{ea}^2 \; var(q_{e1}) \Big] \\
cov(\mu_{bp}, v_a) &= \frac{1}{\tau_{vp}^a \; \tau_v} \big[(\beta_1 - \gamma_1 K_a)\gamma_1 \tau_z + (\gamma_1 - \beta_1 K_a)\beta_1 \tau_z + \omega_a \beta_{e1} \tau_{ze} K_{ea} \big] \\
cov(\epsilon_{\mathcal{I}_2^B}, \mu_{bp}) &= \frac{1}{\tau_{v|\mathcal{I}_2^B} \; \tau_{vp}^b} \big[(\beta_1 - \gamma_1 K_a)\gamma_1 \tau_z + (\gamma_1 - \beta_1 K_a)\beta_1 \tau_z + \omega_a \beta_{e1} \tau_{ze} K_{ea} \big] \\
cov(\lambda_{a,2} q_{a,2}, \lambda_{b,2} q_{b,2}) &= -\frac{4}{9} IR_B \frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}} (cov(v_a, \mu_{bp}) + cov(v_b, \mu_{ap})) + \frac{4}{9} IR_B^2 cov(\mu_{ap}, \mu_{bp}) - \\
&\quad \frac{2}{9} IR_B \frac{\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}}} (cov(\mu_{bp}, \epsilon_{\mathcal{I}_2^B}) + cov(\mu_{ap}, \epsilon_{\mathcal{I}_2^A}))
\end{aligned}
$$

Hence, the covariance between the prices depends on the variance and covariance of the order flows in the t = 1. These are given by:

$$
\begin{aligned}
var(q_{a1}) &= (\beta_1^2 + \gamma_1^2)\tau_v^{-1} + \tau_z^{-1} \\
var(q_{b1}) &= (\beta_1^2 + \gamma_1^2)\tau_v^{-1} + \tau_z^{-1} \\
var(q_{e1}) &= \beta_{e1}^2(\omega_a^2 + \omega_b^2)\tau_v^{-1} + \tau_{ze}^{-1} \\
cov(q_{a1}, q_{b1}) &= 2(\beta_1 \gamma_1)\tau_v^{-1} \\
cov(q_{a1}, q_{e1}) &= (\omega_a \beta_1 \beta_{e1} + \omega_b \gamma_1 \beta_{e1})\tau_v^{-1} \\
cov(q_{b1}, q_{e1}) &= (\omega_b \beta_1 \beta_{e1} + \omega_a \gamma_1 \beta_{e1})\tau_v^{-1}
\end{aligned}
$$

Inserting these quantities into the previous equations yields the covariance between the second-period prices. Note that $cov(P_{a,2}^{ETF}, P_{b,2}^{ETF}) \neq 0$ in general.

Proof of Proposition 4

First, consider the market without an ETF. From Proposition 1 it follows that:

$$
\begin{aligned}
P_{a,1} &= \lambda_{a,1}\; q_{a,1} \\
&= \lambda_{a,1}\; (\beta^{A}_{a,1} v_a + z_{a,1})
\end{aligned}
$$

Hence, the covariance between $P_{a,1}, P_{b,1}$ is given by

$$
\begin{aligned}
cov(P_{a,1}, P_{b,1}) &= \lambda_{a,1}\; \lambda_{b,1}\; cov(q_{a,1}, q_{b,1}) \\
&= \lambda_{a,1}\; \lambda_{b,1}\; \beta^{A}_{a,1}\; \beta^{B}_{b,1}\; cov(v_a, v_b) \\
&= 0
\end{aligned}
$$

where $cov(v_a, v_b) = 0$ by assumption.

Next, let us turn the economy with an ETF. According to Proposition 2, it is

$$
\begin{aligned}
P^{ETF}_{a,1} &= \lambda_{a,1}\; q_{a,1} \\
&= \lambda_{a,1}\; (\beta^{A}_{a,1} v_a + \gamma^{B}_{a,1} v_b + z_{a,1})
\end{aligned}
$$

which yields:

$$
\begin{aligned}
cov(P^{ETF}_{a,1}, P^{ETF}_{b,1}) &= \lambda_{a,1}\; \lambda_{b,1}\; cov(q_{a,1}, q_{b,1}) \\
&= \lambda_{a,1}\; \lambda_{b,1}\; cov(\beta^{A}_{a,1} v_a + \gamma^{B}_{a,1} v_b + z_{a,1},\; \gamma^{A}_{b,1} v_b + \beta^{B}_{b,1} v_b + z_{b,1}) \\
&= \lambda_{a,1}\; \lambda_{b,1}\; (\beta^{A}_{a,1}\gamma^{A}_{b,1} + \beta^{B}_{b,1}\gamma^{B}_{a,1})\; \tau_v^{-1} \\
&> 0
\end{aligned}
$$

where the last inequality follows from constraints $\beta^{A}_{a,1} > 0, \beta^{B}_{b,1} > 0, \gamma^{A}_{b,1} > 0, \gamma^{B}_{a,1} > 0$, which are due to the second-order conditions given in Proposition 2.

Proof of Corollary 4

To analyse the effect of a change in v_b on the price of the asset v_a in the underlying asset market, let us analyse the partial derivatives of the prices wrt. v_b. First, consider the economy without the ETF. From Proposition 1, it is immediate to verify that:

$$\frac{\partial P_{a,1}}{\partial v_b} = 0$$
$$\frac{\partial P_{a,2}}{\partial v_b} = 0$$

For the sake of completeness, note that the price of the ETF as well as the price of v_b adjust if v_b changes.

Next, consider the economy with an ETF. From Proposition 2, we know that:

$$P_{a,1}^{ETF} = \lambda_{a,1}\ (\beta_{a,1}^{A} v_a + \gamma_{a,1}^{B} v_b + z_{a,1})$$
$$\mu_{ap} = \frac{1}{\tau_{vp}^{a}} * \big((\beta_{a,1}^{A} q_{a,1} + \gamma_{b,1}^{A} q_{b,1})\tau_z - (\beta_{b,1}^{B} q_{b,1} + \gamma_{a,1}^{B} q_{a,1})\ \tau_z\ K_a + q_{e1}\tau_{ze}K_{ea}\big)$$
$$P_{a,2}^{ETF} = IR_B\ \mu_{ap} + \lambda_{a,2}\big(\beta_{a,2}^{A}(v_a - \mu_{ap}) + \gamma_{a,2}^{B}(\mathbb{E}(v_a|\mathcal{I}_2^B) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}_2^B}})\mu_{ap}) + z_{a,2}\big)$$

Note that:

$$\frac{\partial \mathbb{E}(v_a|\mathcal{I}_2^B)}{\partial v_b} = 0$$
$$\frac{\partial q_{a,1}}{\partial v_b} = \gamma_{a,1}^{B}$$
$$\frac{\partial q_{b,1}}{\partial v_b} = \beta_{b,1}^{B}$$
$$\frac{\partial q_{e1}}{\partial v_b} = \beta_{e1}^{B}$$

Using that $\gamma^B_{a,2} \cdot (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^B_2}}) = \beta^A_{a,2} = \frac{1}{3\lambda_{a,2}} IR_B$, it follows that:

$$\begin{aligned}
\frac{\partial P^{ETF}_{a,1}}{\partial v_b} &= \lambda_{a,1}\ \gamma^B_{a,1} \\
\frac{\partial \mu_{ap}}{\partial v_b} &= \frac{1}{\tau^a_{vp}} * \big((\beta^A_{a,1}\gamma^B_{a,1} + \gamma^A_{b,1}\beta^B_{b,1})\tau_z - (\beta^{B^2}_{b,1} + \gamma^{B^2}_{a,1})\ \tau_z\ K_a + \beta^B_{e1}\tau_{ze}K_{ea}\big) \\
\frac{\partial P^{ETF}_{a,2}}{\partial v_b} &= IR_B\ \frac{\partial \mu_{ap}}{\partial v_b} + \lambda_{a,2}\ (-\beta^A_{a,2}\ \frac{\partial \mu_{ap}}{\partial v_b} - \beta^A_{a,2}\frac{\partial \mu_{ap}}{\partial v_b}) \\
&= \frac{1}{3} IR_B \frac{\partial \mu_{ap}}{\partial v_b}
\end{aligned}$$

Using the symmetry of the equilibrium and writing $\beta^A_{a,1} = \beta^B_{b,1} = \beta_1$ and $\gamma^A_{b,1} = \gamma^B_{a,1} = \gamma_1$, we can simplify

$$\frac{\partial \mu_{ap}}{\partial v_b} = \frac{1}{\tau^a_{vp}} * \big(2\beta_1\gamma_1\tau_z - (\beta^2_1 + \gamma^2_1)\ \tau_z\ K_a + \omega_b\beta_{e1}\tau_{ze}K_{ea}\big)$$

Overall, note that, in equilibrium, a change in v_b affects the price of asset a in all periods. Moreover, we see that $\frac{\partial P^{ETF}_{a,1}}{\partial v_b} \neq 0$ due to cross-market trading, while $\frac{\partial P^{ETF}_{a,2}}{\partial v_b} \neq 0$ due to the impact of v_b on μ_{ap}, hence due to cross-market learning by the market maker.

Proof of Proposition 5

Asset speculator A expects to trade

$$\begin{aligned}
\mathbb{E}(x^A_{b,2}|v_a) &= \mathbb{E}\big(\gamma^A_{b,2}(\mathbb{E}(v_b|\mathcal{I}^A_2) - (1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\mu_{bp})|v_a\big) \\
&= -\gamma^A_{b,2}(1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\mathbb{E}(\mu_{bp})|v_a) \\
&= -\gamma^A_{b,2}(1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\frac{1}{\tau^b_{vp}}\big((\beta^B_{b,1}x^A_{b,1} + \gamma^B_{a,1}x^A_{a,1})\tau_z - (\gamma^A_{b,1}x^A_{b,1} + \beta^A_{a,1}x^A_{a,1})\tau_z K_b + \beta^A_{e1}v_a\tau_{ze}K_{eb}\big)
\end{aligned}$$

Let us use symmetry and rewriting $\beta^A_{a,1} = \beta^B_{b,1} = \beta_1$ and $\gamma^A_{b,1} = \gamma^B_{a,1} = \gamma_1$ yields:

$$\begin{aligned}
\mathbb{E}(x^A_{b,2}|v_a) &= -\gamma^A_{b,2}(1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\frac{1}{\tau^b_{vp}}\big((\beta_1 - \gamma_1 K_b)x^A_{b,1}\tau_z + (\gamma_1 - \beta_1 K_b)x^A_{a,1}\tau_z + \omega_a\beta_{e1}v_a\tau_{ze}K_{eb}\big) \\
&= -\gamma^A_{b,2}(1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\frac{1}{\tau^b_{vp}}\big((\beta_1 - \gamma_1 K_b)\gamma_1\tau_z + (\gamma_1 - \beta_1 K_b)\beta_1\tau_z + \omega_a\beta_{e1}\tau_{ze}K_{eb}\big)\ v_a \\
&= -\gamma^A_{b,2}(1 + \frac{\tau_v}{\tau_{v|\mathcal{I}^A_2}})\frac{\tau_z}{\tau^b_{vp}}\big(2\beta_1\gamma_1 - (\beta_1^2 + \gamma_1^2)K_b + \omega_a\beta_{e1}\eta^{-2}K_{eb}\big)\ v_a
\end{aligned}$$

Except for $\big(2\beta_1\gamma_1 - (\beta_1^2 + \gamma_1^2)K_b + \omega_a\beta_{e1}\eta^{-2}K_{eb}\big)$, all factors are positive. Hence, asset speculator A expects to reverse her position taken in t = 1 iff

$$\begin{aligned}
\big(2\beta_1\gamma_1 - (\beta_1^2 + \gamma_1^2)K_b + \omega_a\beta_{e1}\eta^{-2}K_{eb}\big) &> 0 \\
2\beta_1\gamma_1(1 - K_b) - (\beta_1 - \gamma_1)^2K_b + \omega_a\beta_{e1}\eta^{-2}K_{eb}\big) &> 0 \\
2\beta_1\gamma_1(1 - K_b) + \omega_a\beta_{e1}\eta^{-2}K_{eb} &> (\beta_1 - \gamma_1)^2K_b
\end{aligned}$$

I demonstrate numerically that this condition holds for almost all exogenous parameter values.

D Expected profits in both models

First, let us begin with the expected profit of an informed speculator in an economy without an ETF.

It is:

$$\mathbb{E}(\pi) = \mathbb{E}\big(x^A_{a1}(v_a - P_{a1}) + x^A_{a2}(v_a - P_{a2})\big)$$

Using the following equalities

$$\begin{aligned}
x^A_{a1} &= \beta^A_{a1} v_a \\
P_{a1} &= \lambda_{a1}\beta^A_{a1} v_a + \lambda_{a1} z_{a1} \\
x^A_{a2} &= \beta^A_{a2}(v_a - P_{a1}) \\
P_{a2} &= \kappa_{a2} + \lambda_{a2} q_{a2} \\
&= \frac{1}{2} P_{a1} + \frac{1}{2} v_{a1} + \lambda_{a2} z_{a2}
\end{aligned}$$

then yields

$$\begin{aligned}
\mathbb{E}(\pi) &= \mathbb{E}\big(\beta^A_{a1} v_a (v_a - \lambda_{a1}\beta^A_{a1} v_a - \lambda_{a1} z_{a1}) + \beta^A_{a2}(v_a - P_{a1})(\frac{1}{2}(v_a - P_{a1}) + \lambda_{a2} z_a 2)\big) \\
&= \mathbb{E}\big(\beta^A_{a1}(1 - \lambda_{a2}\beta^A_{a1}) v_a^2\big) + \mathbb{E}\big(\frac{1}{2}\beta^A_{a2}(v_a - P_{a1})^2\big) \\
&= \beta^A_{a1}(1 - \lambda_{a2}\beta^A_{a1})\ var(v) + \frac{1}{2}\beta^A_{a2}\ var(v_a - P_{a1}) \\
&= \beta^A_{a1}(1 - \lambda_{a2}\beta^A_{a1})\ var(v) + \frac{1}{2}\beta^A_{a2}\ \big(var(v)(1 - \beta^A_{a1}\lambda_{a1})^2 + \lambda^2_{a1} var(z)\big)
\end{aligned}$$

Next, let us turn to the expected profit in an economy with an ETF. Let us use the symmetry of the equilibrium and denote $\beta^A_{a1} = \beta^B_{b1} = \beta_a, \gamma^B_{a1} = \gamma^A_{b1} = \gamma_a, \beta^A_{e1} = \omega_a\beta_e, \beta^B_{e1} =$

$\omega_b \beta_e, \beta^A_{a2} = \beta^B_{b2} = \beta_2, \gamma^B_{a2} = \gamma^A_{b2} = \gamma_2$. I begin by defining

$$\mathbb{E}(\pi^{ETF}) = \mathbb{E}\big(x^A_{a1}(v_a - P_{a1}) + x^A_{b1}(v_b - P_{b1}) + x^A_{a2}(v_a - P_{a2}) + x^A_{b2}(v_b - P_{b2})\big)$$

and consider the individual parts going forwards. Analogous to the economy without an ETF, it is:

$$\mathbb{E}\big(x^A_{a1}(v_a - P_{a1})\big) = \beta_1(1 - \lambda_1\beta_1)\ var(v)$$

Next,

$$\begin{aligned}\mathbb{E}\big(x^A_{b1}(v_b - P_{b1})\big) &= \mathbb{E}\big(\gamma_1(1 - \lambda_1\beta_1)\ v_b\ v_a - x^A_{b1}\lambda_1 z_{b1} - \lambda_{b1}\gamma_1^2\ v_a^2\big) \\ &= -\lambda_{b1}\gamma_1^2\ var(v)\end{aligned}$$

Consequently, the expected profit of trading in market $j \neq i$ in period 1 is negative. Next, consider the continuation profits in t = 2. Using the same steps as before, it is:

$$\begin{aligned}\mathbb{E}\big(x^A_{a2}(v_a - P_{a2})\big) &= \mathbb{E}\big(\beta_2\frac{1}{3}(v_a - \mu_{ap})^2\big) \\ &= \frac{1}{3}IR_B\beta_2\ var(v_a - \mu_{ap})\end{aligned}$$

Using standard properties of the variance of a sum, I find that:

$$\begin{aligned}var(v_a - \mu_{ap}) = var(v) - \frac{2}{\tau^a_{vp}} \cdot var(v) \cdot \big[(\beta_2^2 + \gamma_2^2)\tau_z - 2\beta_2\gamma_2 K_a\tau_z + \omega_a\beta_e\tau_{ze}K_{ei}\big] + \\ \Big(\frac{1}{\tau^a_{vp}}\Big)^2 \cdot \Big[\big((\beta_1 - \gamma_1 K_a)^2 + (\gamma_1 - \beta_1 K_a)^2\big)\tau_z^2 var(q_{a1}) + K_{ei}^2\tau_{ze}^2 var(q_{e1}) + \\ 2(\beta_1 - \gamma_1 K_a)(\gamma_1 - \beta_1 K_a)\tau_z^2 cov(q_{a1}, q_{b1}) + 2(\beta_1\gamma_1 K_a)\tau_z\tau_{ze}K_{ei}cov(q_{e1}, q_{a1}) + \\ 2(\gamma_1 - \beta_1 K_a)\tau_z\tau_{ze}K_{ei}cov(q_{e1}, q_{b1})\Big]\end{aligned}$$

Finally, the variances and covariances of the order flows are given by:

$$
\begin{aligned}
var(q_{a1}) = var(q_{b1}) &= (\beta_1^2 + \gamma_1^2)\ var(v) + var(z) \\
var(q_{e1}) &= \beta_e^2(\omega_a^2 + \omega_b^2)\ var(v) + \eta^2 var(z) \\
cov(q_{a1}, q_{b1}) &= 2\beta_1\gamma_1 var(v) \\
cov(q_{a1}, q_{e1}) &= (\omega_a\beta_e\beta_1 + \omega_b\beta_e\gamma_1)var(v) \\
cov(q_{a1}, q_{e1}) &= (\omega_b\beta_e\beta_1 + \omega_a\beta_e\gamma_1)var(v)
\end{aligned}
$$

Inserting these quantities yields the expression for $\mathbb{E}\big(x_{a2}^A(v_a - P_{a2})\big)$. The last summand of the expected profit, given by $\mathbb{E}\big(x_{b2}^A(v_b - P_{b2})\big)$, can be found in the same way. It holds:

$$
\begin{aligned}
\mathbb{E}\big(x_{b2}^A(v_b - P_{b2})\big) = \frac{1}{3}\frac{1}{1 + \frac{1}{3}\frac{\tau_v}{\tau_{v|\mathcal{I}_2^A}}}\gamma_1 \Bigg[&(1 - (\frac{\tau_v}{\tau_{|\mathcal{I}_2^A}})^2)\ var(v) + (1 + \frac{\tau_v}{\tau_{|\mathcal{I}_2^A}})^2\ var(\mu_{bp}) - (\frac{\tau_v}{\tau_{|\mathcal{I}_2^A}})^2\ var(\epsilon_{\mathcal{I}_2^A}) - \\
&(1 + \frac{\tau_v}{\tau_{|\mathcal{I}_2^A}})\ cov(\mu_{bp}, v_b)\Bigg]
\end{aligned}
$$

Inserting

$$
\begin{aligned}
var(\mu_{bp}) = (\frac{1}{\tau_{vp}^b})^2 \Bigg[&\Big((\beta_1 - \gamma_1 K_b)^2 + (\gamma_a - \beta_1 K_b)^2\Big)\tau_z^2\ var(q_{b1}) + K_{ej}^2\tau_{ze}^2 var(q_{e1}) + \\
&2(\beta_1 - \gamma_1 K_a)(\gamma_1 - \beta_1 K_a)\tau_z^2 cov(q_{a1}, q_{b1}) + 2(\beta_1\gamma_1 K_a)\tau_z\tau_{ze}K_{ei}cov(q_{e1}, q_{b1}) + \\
&2(\gamma_1 - \beta_1 K_a)\tau_z\tau_{ze}K_{ei}cov(q_{e1}, q_{a1})\Bigg] \\
var(\epsilon_{\mathcal{I}_2^A}) = &\frac{1}{\tau_v^2}((\gamma_1^2 + \beta_1^2)\tau_z + \omega_a^2\beta_e^2\tau_{ze}) \\
cov(\mu_{bp}, v_b) = &\frac{1}{\tau_{vp}^b} \cdot var(v) \cdot \big[(\beta_1^2 + \gamma_1^2)\tau_z - 2\beta_1\gamma_1 K_b\tau_z + \omega_b\beta_e\tau_{ze}K_{ej}\big]
\end{aligned}
$$

yields the expression for the expected profit of the speculator in an economy with an ETF.